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Large-eddy simulation of the turbulent near wake behind a circular cylinder: Reynolds number effect

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ABSTRACT

The purpose of the present work is <!–<query>The highlights are in an incorrect format. Hence they have been deleted. Please refer the online instructions: http://www.elsevier.com/highlights and provide 3-5 bullet points.</query>–>to study the effect of the Reynolds number on the near-wake structure and separating shear layers behind a circular cylinder. Three-dimensional unsteady large-eddy simulation is carried out and two different subgrid scale models are applied in order to evaluate the turbulent wake reasonably. The Reynolds number based on the free-stream velocity and the cylinder diameter is ranging from Re = 5500–41,300, corresponding to the full development of the shear-layer instability in the intermediate subcritical flow regime. For a complete validation of this numerical study, hydrodynamic bulk coefficients are computed and compared to experimental measurements and numerical studies in the literature. Special focus is made on the variations of both the large-scale near-wake structure and the small-scale shear-layer instability with increasing Reynolds numbers. The present numerical study clearly shows the broadband nature of the shear-layer instability as well as the dependence of the shear-layer frequency especially on the high Reynolds numbers.

1. Introduction

The practical simple geometry of the circular cylinder is one of the most commonly used structural members in ocean and offshore engineering, but the flow behind the simple geometry especially at higher Reynolds numbers involves extremely complex flow features such as freely separating shear layers, small-scale vortices in the thin boundary layers and large-scale vortex motion in the wake. As a consequence, the flow over a circular cylinder is a multi-scale problem between Kármán instability in the near wake and Kelvin–Helmholtz like instability in the thin shear layers.

The investigation of the near wake behind a cylinder, therefore, has been one of the most interesting research subjects during decades: see [2,28,14,26,39,42,31,8,24] and references therein. The freely separating shear layers of the cylinder rolls up and forms the Kármán vortices. Small-scale vortices are observed in the shear layers, called Kelvin–Helmholtz or shear-layer vortices, prior to the formation of the Kármán vortices. The Reynolds number seems to have a significant influence on the characteristics of the shear layer and the vortex formation zone [30]. The pioneering study of the small-scale vortices on the onset of the shear-layer transition at a certain Reynolds number Re_{crit} and its predominant shear-layer frequency f_{SL} was conducted by Bloor [5]. Bloor [5] suggested that the instability sets in at $\mathrm{Re}_{crit} \approx 1300$ and the shear-layer frequency to the Kármán frequency ratio f_{SL}/f_K is scaled with $\mathrm{Re}^{0.5}$ power law. The variation of the normalized shear-layer frequency as well as Re_{crit} has been intensively studied in the previous experiments: see [37,25,9] and references therein. Prasad and Williamson [29] conducted evaluation of previous data and suggested the relationship $f_{SL}/f_K = 0.0235 \times \mathrm{Re}^{0.67}$ between Re = 1200 and 45,000. More recently, Rajagopalan and Antonia [30] provided a scale exponent of 0.65, which is in agreement with the $\mathrm{Re}^{0.67}$ power law suggested by Prasad and Williamson [29]. Thompson and Hourigan [33] provided an excellent review and data concerning on this phenomenon. Their results showed $\mathrm{Re}^{0.57}$ dependence for Re < 4000 and $\mathrm{Re}^{0.52}$ for Re > 10,000.

A number of 2-D and 3-D numerical simulations of the cylinder flow has been conducted in the intermediate subcritical regime at Re = 1000–40,000 by various numerical methods, mostly 2-D Reynolds-averaged Navier–Stokes (RANS) at higher Reynolds numbers. In the intermediate regime, the flow at Re = 3900 have been the most extensively studied in order to predict the flow features behind 3-D cylinder in a general point view, see e.g. [3,6,22]. Blackburn et al. [4] stressed the significance of three-dimensionality of the cylinder flow and also suggested that 3-D simulation is required in order to reproduce experimental observations, even at lower Reynolds numbers. Breuer [7] studied the cylinder flow at Re = 140,000 in order to evaluate the applicability of 3-D LES in the practical high Reynolds number. Their results of the main flow

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parameters and flow statistics showed satisfactory agreement with the experimental data.

Jordan [15] conducted 3-D large-eddy simulation (LES) study with particular focus on the shear-layer instability at higher Reynolds number of 8000 and their results of the shear-layer frequency showed good agreement with previous experimental data. They noted that the dynamic SGS model is not a major contributor to the predicted shear-layer characteristics. Dong et al. [11] studied the Reynolds number effect between Re = 3900 and 10,000 using direct numerical simulation (DNS) approach and a number of flow statistics were calculated reasonably compared with their PIV experimental measurements. They captured the broadband nature of the shear-layer frequency at Re = 3900, but the frequency at higher Reynolds number of 10,000 exhibited the plateau feature in their power spectra. The DNS study clearly showed that the predominant frequencies of shear-layer vortices follow $\mathrm{Re}^{0.67}$ power law as increasing the Reynolds number from Re = 3900 to 10,000. [40] conducted 3-D LES study at Re = 3900,10,000 and 20,000, based on the eddy-viscosity type subgrid scale (SGS) model. The forces and pressure coefficients are predicted reasonably well. Their instantaneous vorticity contours on a high resolution of 3.8 million cells at Re = 20,000 showed the instability visually, but no spectral analysis for the shear-layer frequency was reported.

In the present study, based on the DNS results at Re = 3900–10,000 from [11], the effect of Reynolds number on the near-wake structures and shear-layer frequency of a circular cylinder is investigated at the extended Reynolds numbers Re = 5500–41,300, in which the shear-layer instability is fully developing in the subcritical flow regime. The 3-D simulations in the frame of LES are conducted with two different subgrid scale models in order to evaluate the flow field suitably at each Reynolds number. We particularly focus on the comparison of the mean flow statistics and shear-layer frequencies with increasing Reynolds number between the predicted results and available experimental/DNS data.

2. Computational modelling

2.1. Basic equations and turbulence model

The 3-D unsteady flow around a cylinder is modelled in the frame of LES approach. The LES equations are derived from the classical time-dependent filtered Navier–Stokes equations:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) + \tau_{ij}\right], \tag{2}$$

where ν is the kinematic viscosity and $\bar{u}$ and $\bar{p}$ are the filtered velocity and pressure respectively. In the spatially filtered Navier–Stokes equations, the SGS stress $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ is to be approximated using a SGS model to get a full solution for the filtered equations. The SGS stress τ_{ij} is expressed according to the Boussinesq's approximation by the adoption of a turbulent eddy viscosity ν_t:

$$\tau_{ij} - \frac{2}{3}k_t\delta_{ij} = -2\nu_t\left(\bar{S}_{ij} - \frac{1}{3}\bar{S}_{kk}\delta_{ij}\right), \tag{3}$$

$$\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right), \tag{4}$$

where $\bar{S}_{ij}$ is the rate of strain tensor computed from the resolved scales and k_t is the SGS turbulent kinetic energy. The algebraic Smagorinsky (SMG) and dynamic k-equation (DTKE) eddy viscosity SGS models are employed in the present study. A comprehensive review of the SGS models is explained in great detail in Ref. [13] and a brief description is given in the following.

Table 1
Summary of numerical set-up.

Re	max. y^+	L_z	$\Delta tU/D$	Flow data[a]	Cores[b] /CPU time
5500	≈1.0	πD	0.002	above 25 cycles	108/168 h
41,300	≈1.0	πD	0.0005	above 25 cycles	96/360 h

[a] Compiling flow data over at least 25 shedding cycles for the statistic convergence, used to compute main wake parameters.

[b] Using a parallel processing on 2.6 GHz Intel Xeon core and 4 GB of memory.

The turbulent eddy viscosity ν_t and SGS turbulent kinetic energy k_t are calculated explicitly in the Smagorinsky model [32], resulting from the local equilibrium balance between shear production and dissipation in the turbulent kinetic energy (TKE) equation [41]:

$$\nu_t = (C_s\Delta)^2\left(2\bar{S}_{ij}\bar{S}_{ij}\right)^{0.5} = \frac{C_k^{0.75}}{C_\epsilon^{0.25}}\Delta^2\left|\bar{S}\right|, \tag{5}$$

$$k_t = \frac{2C_k}{C_\epsilon}\Delta^2\left|\bar{S}\right|^2, \tag{6}$$

where Δ is the grid filter width estimated as the cubic root of the control volume (CV). The dimensionless model coefficients are assigned as the values of $C_\epsilon = 1.0$ and $C_k = 0.05$ [13], yielding the well-known Smagorinsky model coefficient of $C_s = 0.1$ which is a practical value in the cylinder flow for SMG model [6]. In DTKE [23], the SGS kinetic energy is estimated by solving TKE equation with no local equilibrium assumption:

$$\frac{\partial k_t}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{u}_j k_t\right) = P + \frac{\partial}{\partial x_j}\left[(\nu + \nu_t)\frac{\partial k_t}{\partial x_j}\right] - \epsilon, \tag{7}$$

$$P = -2\nu_t\bar{S}_{ij}\bar{S}_{ij}, \quad \epsilon = C_\epsilon k_t^{3/2}\Delta^{-1},$$

where the SGS viscosity is given by $\nu_t = C_k\Delta k^{0.5}$. The model coefficients C_ϵ and C_k are dynamically computed as part of solution based on the Germano identity with test filter $\hat{\Delta} = 2\Delta$ by the least square minimization procedure proposed by Lilly [20]. For equilibrium value of the dissipation term, $C_\epsilon = 1.0$ is reasonably assumed in the present study.

A wall damping formulation suggested by Van Driest [36] is implemented based on the successful computation of the cylinder flow, see e.g. [1]. The wall damping model is derived by changing filter width Δ and combined with any SGS model, which is denoted as +WD with the name of the SGS model.

2.2. Computational domain and numerical method

A curvilinear O-type orthogonal grid is generated, which have been widely used in the numerical simulations of the cylinder flow [21,6,1]. The hexahedral grid in the x–y plane is exponentially stretched in the radial direction r and is uniformly spaced in the circumferential direction θ. The entire computational domain has a radial extension of 15 cylinder diameters 15D from the cylinder centre and the spanwise length L_z is πD for all simulation cases. The dimensions of the solution domain including the cylinder aspect ratio L_z/D are chosen based on careful previous 3-D LES studies of the cylinder flow [7,40,17]. The resulting nondimensional distance to the cylinder wall is kept as $y^+ \approx 1.0$ in whole simulation times to resolve the boundary layer on the cylinder surface. Table 1 summarizes some key characteristics at the considered Reynolds numbers.

Uniform flow U is imposed at the inflow without the disturbance level and the Neumann conditions are applied at the outlet boundary. The periodicity of the flow are assumed at the spanwise ends of