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Extended Boussinesq equations for waves in porous media

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ABSTRACT

We develop an extended Boussinesq model to predict the propagation of waves in porous media. The inertial and drag resistances are taken into account in the developed model. When these resistances are neglected, the developed equations are reduced to the extended Boussinesq equations of Madsen and Sørensen (1992) [Coastal Eng. 18, 183–204] for non-porous media The model can be applied for non-porous media as well as porous media because it does not need any matching condition at the interface between the porous and non-porous media. The developed model is verified by comparing numerical solutions to experimental data with several cases of solitary waves or sinusoidal waves interacting with a porous breakwater in one- and two-dimensional domains.

1. Introduction

Rubble-mound breakwaters have been constructed worldwide for a long time. They are easy to build, safe against severe wave climate, and efficient to protect facilities behind the breakwater. Stones or concrete blocks are placed in a trapezoidal shape with a slope of 1:2 to 1:3 on sea side and of 1:1 to 1:1.5 on land side. Armor units can be placed further on the sea side to dissipate incoming wave energy. Wave energy dissipation through the porous breakwater occurs dominantly via the turbulence of high speed waters and also by the friction of waters through porous media. These are called the turbulent and laminar drag resistances, respectively. The energy dissipation also occurs due to unsteadiness of water waves in porous media so-called inertial resistance. In the 1970s, the energy dissipation of water waves in porous breakwaters was analytically investigated by several researchers (Sollitt and Cross, 1972; Madsen, 1974) considering both the inertial resistance and the drag resistance of the Forchheimer (1901) type. They assumed that water waves are linear and are normally incident to the breakwater, water depth is constant and the breakwater is in rectangular shape. In the 1990s, the energy dissipation of water waves in porous breakwaters could be obtained for waves obliquely incident to the breakwater and also for varying water depth using the mild-slope equations (Izumiya, 1990). Also, researchers were concerned about the energy dissipation of water waves with two layers, i.e., upper non-porous layer and lower porous layer such as water waves over a submerged porous breakwater (Izumiya, 1990; Cruz et al., 1997; Hsiao et al., 2002) or a porous bed (Mase et al., 1995; Corvaro et al., 2010, 2014a; b; Torres-Freyermuth et al., 2017). The solution could be obtained using the mild-slope equations (Izumiya, 1990; Mase et al., 1995), the Boussinesq equations (Flaten and Rygg, 1991; Cruz et al., 1997; Hsiao et al., 2002), the Navier-Stokes equations (Torres-Freyermuth et al., 2017) or through physical experiments (Corvaro et al., 2010, 2014a; b). Since the 1970s, the energy dissipation of water waves in porous breakwaters could be obtained using the Boussinesq equations (Abbott et al., 1978; Madsen and Warren, 1984; Liu and Wen, 1997). These equations consider the drag resistance of the Forchheimer type but not the inertial resistance. When the drag resistance is neglected, these equations are reduced to the equations of Peregrine (1967). When applied to porous breakwaters, Liu and Wen used two different types of Boussinesq equations, one for non-porous media and the other for porous media, and, at the interface, they used matching conditions. For waves normally incident on a porous breakwater in front of a wall, Madsen (1983) compared reflection coefficients between the linear analytical solution and the numerical solution of the Boussinesq equations considering the drag resistance and neglecting the inertial resistance. He found that the two solutions are close to each other for linear waves but these are different for nonlinear waves. In engineering applications, people often use the Boussinesq equations because these equations yield solutions for real waves such as nonlinear and random waves whereas the mild-slope equations yield solutions only for linear and regular waves. The Boussinesq equations of Peregrine (1967) can be applied only in shallow water but not in deep water due to weakly dispersive condition. Since the 1990s, the

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Boussinesq equations have been developed to be applied even in deep water by including dispersive terms in the momentum equations (Madsen and Sørensen, 1992) or using particle velocities at a certain elevation (Nwogu, 1993). These Boussinesq equations mentioned before have been developed on the assumption of irrotational flow and are classified as the first class of the Boussinesq models by Brocchini (2013). Recently, due to the development of computer technology, numerical models were developed to directly solve the Navier-Stokes equations for waves in porous breakwaters such as NS3 (Mayer et al., 1998), COBRA (Liu et al., 1999) and CADMAS-SURF (CDIT, 2001). These models can predict not only the run-up of waves on the breakwater but also the overtopping which cannot be predicted by the depth-integrated models such as the Boussinesq equations and the mild-slope equations. However, it takes so long time to solve the Navier-Stokes equations in three-dimensional phenomena such as refraction and diffraction. Thus, these models are not used yet in applying to real situations.

In the present study, we develop extended Boussinesq equations for porous media considering both the inertial resistance and the drag resistance of the Forchheimer type. When these resistances are neglected, the equations are reduced to the equations of Madsen and Sørensen (1992). In section 2, we develop the Boussinesq equations for porous media in shallow water and then the developed equations are extended to deeper water by adding dispersive terms in the momentum equations. Also, the dispersion relations are investigated in detail and the effects of the inertial and drag resistances are compared each other. In section 3, we conduct numerical experiments for solitary waves propagating through a porous breakwater. A solution of solitary waves for non-porous water is derived and used as an initial condition. Numerical solutions are compared to hydraulic experimental data for horizontally one- and two-dimensional domains. In section 4, we summarize our research works.

2. Development of Boussinesq equations for waves in porous media

2.1. Boundary value problems

The variables and domain of interest are shown in Fig. 1. The domain covers not only porous media (porosity $\lambda < 1$) but also non-porous media (porosity $\lambda = 1$). The water surface is displaced by η from still water. The bottom is located at $z = -h$ from the still water. In order to develop the extended Boussinesq equations for waves in porous media, we set up boundary value problems for waves inside porous media. Since the porosity is uniform, the continuity equation in porous media is given by

$$\nabla_3 \cdot \mathbf{U} = 0 \tag{1}$$

where $\mathbf{U} = (u, v, w)$ is the seepage velocity vector and $\nabla_3 \equiv (\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$ is the gradient operator. The momentum equation in porous media is given by

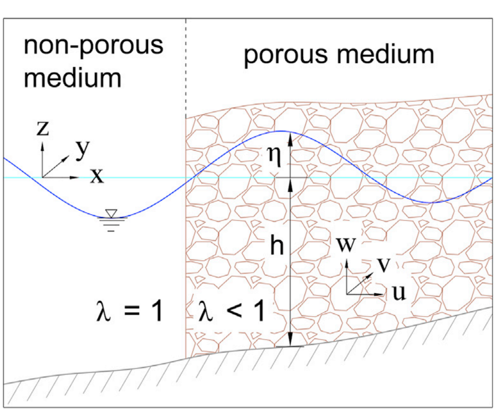

Fig. 1. Definition of variables in interested domain.

$$\lambda \frac{d\mathbf{U}}{dt} + \frac{1}{\rho}\nabla_3 (p + \rho g z) + D + I = 0 \tag{2}$$

where λ is the porosity, p is the pore pressure, ρ is water density, g is the gravitational acceleration, D is the drag resistance term, and I is the inertial resistance term. It should be noted that the continuity equation (1) and the momentum equation (2) are expressed in terms of the seepage velocity instead of the discharge velocity.

There are several ways to define the drag resistance term. Ergun (1952) defined the drag resistance term in the Forchheimer (1901) type using a volume-averaged discharge velocity $\mathbf{U}'(= \lambda \mathbf{U})$. In the present study, we use Ergun's definition of D in terms of the seepage velocity $\mathbf{U}$ instead:

$$\begin{aligned} D &\equiv \alpha_l \left(\frac{1-\lambda}{\lambda}\right)^2 \frac{\nu}{d^2}\frac{\mathbf{U}'}{\lambda} + \alpha_t \frac{1-\lambda}{\lambda}\frac{1}{d}\left|\frac{\mathbf{U}'}{\lambda}\right|\frac{\mathbf{U}'}{\lambda} \\ &= \alpha_l \left(\frac{1-\lambda}{\lambda}\right)^2 \frac{\nu}{d^2}\mathbf{U} + \alpha_t \frac{1-\lambda}{\lambda}\frac{1}{d}|\mathbf{U}|\mathbf{U} \end{aligned} \tag{3}$$

where α_l and α_t are coefficients representing the laminar and turbulent flow resistances, respectively, ν is the kinematic viscosity of water, and d is the size of the solid material. Engelund (1953) also used the Forchheimer type to define the drag resistance term:

$$D \equiv \alpha_{lE}\frac{(1-\lambda)^3}{\lambda}\frac{\nu}{d^2}\frac{\mathbf{U}'}{\lambda} + \alpha_{tE}\frac{1-\lambda}{\lambda}\frac{1}{d}\left|\frac{\mathbf{U}'}{\lambda}\right|\frac{\mathbf{U}'}{\lambda} \tag{4}$$

where α_{lE} and α_{tE} are coefficients which represent the laminar and turbulent flow resistances, respectively, recommended by Engelund. The quadratic term in his formula is the same as that in Ergun's formula, but the linear term to be fitted for porous flow in sand is different. Ward (1964) also suggested the drag resistance term as follows:

$$D = \frac{\nu}{K}\mathbf{U}' + \frac{C_f}{\sqrt{K}}|\mathbf{U}'|\mathbf{U}' \tag{5}$$

where K is the intrinsic permeability and C_f is the turbulent resistance coefficient. This drag resistance term does not depend directly on the porosity and the grain size. The mathematical form of Eq. (3) including the parameters λ, ν, and d can be derived from the Navier-Stokes equation (Irmay, 1958) or from the Reynolds equation (Burcharth and Andersen, 1995). Thus, in the present study, we use the definition of the drag resistance term of Ergun.

The inertial resistance term I in Eq. (2) is given by

$$I \equiv (1-\lambda)(1+\kappa)\frac{d\mathbf{U}}{dt} \tag{6}$$

where κ is the added mass coefficient. In unsteady flow, the inertial resistance term is used to account for the divergence and convergence of the streamlines in the presence of solid material. The inertial resistance term should be considered for both of the local and convective accelerations. The added mass coefficient is for the inertial resistance in view of geometrical smoothness of the solid material. In case of no solid material in the media, the porosity is $\lambda = 1$ and the drag and inertial resistance terms are zero.

Several researchers proposed different momentum equations including the drag and inertial resistance terms. In this study, the momentum equation (2) and the inertial resistance terms (6) are used as Madsen (1974) and Cruz et al. (1997). However, Madsen considered the local acceleration term $\partial \mathbf{U}/\partial t$ and neglected the convective acceleration term $\mathbf{U}\cdot\nabla\mathbf{U}$. The drag resistance term given by Eq. (3) is different from that of Cruz et al. who used Ward's term given by Eq. (5).

Substitution of Eqs. (3) and (6) into the momentum equation (2) gives