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Bed shear stress, surface shape and velocity field near the tips of dam-breaks, tsunami and wave runup

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ABSTRACT

An analytical model is presented for the 2DV flow-structure, bed shear stress and surface shape of the tips of dam-break waves, tsunami- or wave run-up. This model differs from previous analytical models, which have taken the usual 'hydraulics approach', describing bed-parallel velocities only and expressing the bed shear stress, τ in terms of a friction factor from steady, uniform flow and an ad-hoc velocity. The 2DV model presented here gives a simple, rational explanation for the fact that the boundary layer is very thin at the contact point. In turn, this explains the measurements of bed shear stresses, which decay with distance s from the tip, or at a fixed point, with time t since passage of the tip. The manner of τ−decay depends on the growth of the boundary layer thickness δ with distance from the tip. The details of this boundary layer growth depend on challenging turbulence features for unsteady, non-uniform flows over rough and usually mobile beds. For illustrative purposes, details are given for the simple example of $\delta = (\nu_t t)^{1/2} = (\nu_t s/c)^{1/2}$, where ν_t is the nominal eddy-viscosity and c is the speed of the tip, assumed to progress with constant form. With this variation of δ, the model gives $\tau \sim t^{-1/2} = (s/c)^{-1/2}$, which is in good qualitative agreement with measurements. Subsequently, this quasi-steady model gives $h \sim s^{1/4}$ for the depth on a horizontal bed, also in good agreement with experiments. The 2DV flow pattern includes surface particles drifting towards the tip and eventually impacting on the bed at the contact point with full forward momentum and large bed-normal velocity. As well as large local bed shear stresses, this tip-flow pattern involves large vertical accelerations and associated large localized pressures, which are likely to be important for the sediment entrainment under the tip. The eddy viscosity is the only tuning parameter in this simple initial model. Based on shear-stress and depth measurements from laboratory settings, with tip propagation speeds of the order 2 m/s, one finds that the required eddy viscosity is in the range 1×10^{-6} - 18×10^{-6} m^2/s increasing with increasing bed roughness in the range from smooth beds to beds of 2.85 mm fixed sand-grains.

1. Introduction

Dam-break waves and the run-up from tsunami and wind waves on beaches are important examples of hazardous, unsteady, non-uniform flows. Hence, a great deal of effort has gone into understanding and modelling them. Early models, eg Ritter (1892) are based on the non-linear shallow water equations and the method of characteristics. They show that an ideal fluid progresses with a very thin, upward concave, sharp front.

However, classical experiments, by Schoklitsch (1917) and Dressler (1954), and more recently O'Donoghue et al. (2010), Sumer et al. (2011) and Spinewine and Capart (2013) show that the front is not sharp but bull-nose shaped (upward convex) with a (near) vertical front, as depicted in Fig. 1.

The bull-nose shape is due to the fact that, in a real fluid, friction becomes dominant, when the depth gets sufficiently small towards the tip. Hence, models with a friction dominated tip region were developed in the 1950s by Dressler (1952, 1954) and Whitham (1955). These models and many that followed, show how the friction modifies the tip from sharp to generally blunt and how the speed of the tip is reduced due to friction.

The reduction of the tip speed and the friction-generated changes to the overall surface shape are usually small, cf Fig. 2.4.2 of Nielsen (2009): Wave runup on smooth plastic is not visibly faster or higher than on a rough, permeable sand bed ($d_{50} \approx 0.5$ mm). Thus, it may be argued that accounting for friction for inundation modelling purposes is peripheral considering other complications like topographical complexity. However, sediment transport is significant near the tip, see eg Figs. 2 and 17 of Spinewine and Capart (2013) and sediment transport models can only be developed on the basis of boundary layer- and shear stress models. – Shear stress predictions are essential for sediment transport modelling.

Early models, eg, Whitham (1955) were based on the bed shear stress under the tip being uniform. This is however not realistic. Time series of directly measured bed shear stresses, e g, Barnes et al. (2009) and Jiang and Baldock (2015) show that, at a given point, τ decays steeply with time after the passage of the tip. The measured decay is not dissimilar from the $t^{-1/2}$-shape for a boundary layer on a plate after abrupt start-up of a laminar flow. Schlichting and Gersten (2017) p 126.

This $t^{-1/2}$-time-dependence is not reconcilable with the uniform stress hypothesis, but indicates that the bed shear stress is a decreasing

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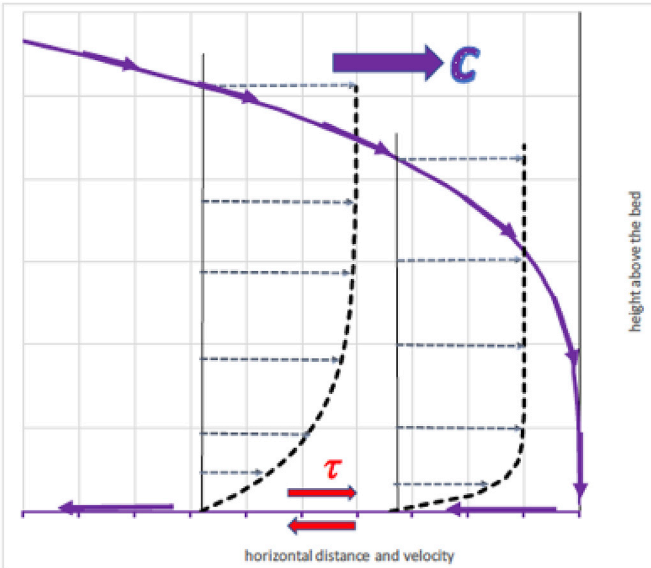

Fig. 1. Swash tip progressing towards the right at speed c.
Dotted curves and arrows indicate velocities relative to the bed.
Arrows along the surface and bottom indicate drift relative to the tip: Forward towards the tip along the surface and backwards corresponding to the no-slip condition at the bed.

function of the distance s from the tip. In related but different scenarios, like the boundary layer developing with distance from the edge along a thin plate, the bed shear stress shows a steeply decaying trend not dissimilar from $\tau \sim s^{-1/2}$, where s is the distance from the edge of the plate. Blasius (1908) gave a laminar solution for this scenario and his solution corresponds to $\tau \sim s^{-1/2}$. The observed behaviour, roughly: $\tau \sim s^{-1/2}$, can be mimicked by hydraulic models with no explicit boundary layer structure and no vertical velocities. This may be achieved by assuming a suitable depth- (Reynolds Number-) dependence of the friction factor in hydraulic style models, e g, Chanson (2009). However, these applications of friction factors from steady, uniform flow to the highly non-uniform and unsteady tip are ad-hoc and the agreement with experiments is fortuitous.

The model, developed here has an explicit, albeit simple, boundary layer structure with which the vertical velocities as well as the bed shear stresses and surface shape can be evaluated, producing theoretical advances over previous analytical models.

We shall develop general relationships between the boundary layer thickness δ, the bed shear stress τ and the surface shape $h(s)$ for a tip, which progresses in a quasi-steady fashion. We find that when $\delta \sim t^{1/2} \sim s^{1/2}$, the momentum equation leads to $\tau \sim t^{-1/2}$ in good agreement with observations. In turn, a quasi-steady force balance, similar to that of Whitham (1955), corrected for the fact that the velocity is not uniform in this model, but varies with distance from the bed, gives $h \sim s^{1/4}$ for a horizontal bed, also in good agreement with experiments.

By comparing with the scarce measurements of surface shapes and bed shear stresses it is found that the eddy viscosity, which is the only tunable parameter in this initial model needs to be in the range $1 \times 10^{-6}\,\mathrm{m}^2/\mathrm{s} < \nu_t < 18 \times 10^{-6}\,\mathrm{m}^2/\mathrm{s}$ for laboratory experiments with tip-propagation velocities of the order 2 m/s. The lower value is found for nominally smooth beds, while the higher value corresponds to beds of fixed sand-grains with $d_{50} = 2.85$ mm.

2. Flow structure and overall dynamics

2.1. Momentum equation for a quasi-steady tip with non-uniform velocity

We consider a scenario like that observed by Baldock et al. (2014) and illustrated in Fig. 1, where fluid is arriving from above to the contact point with full forward velocity c equal to the propagation speed of the tip. Equality of particle velocity at the contact point with the tip propagation speed corresponds to the surface tangent being perpendicular to the bed at the tip. This assumption is justified below, following Equation (13), where it is shown that the surface tangent must indeed be perpendicular to the bed for a wide class of functions describing the bed shear stress' dependence upon the distance from the tip. The no-slip boundary layer thickness is then practically zero at the tip and will grow with distance from the tip. The momentum defect corresponding to this boundary layer growth is 'injected' by the bed shear stress.

In the frame of reference, which follows the steadily progressing tip, illustrated in Fig. 1, the momentum equation (for motion parallel with the bed) is

$$0 = -\tau - \varrho g h \frac{\partial h}{\partial x} - \frac{\partial}{\partial x} \int_{\mathrm{bed}}^{\mathrm{surface}} \varrho (v - c)^2 \mathrm{d}n \tag{1}$$

where x is the coordinate in direction of tip propagation relative to a fixed point on the bed, n is the coordinate perpendicular to the bed and ϱ is the fluid density. As per Fig. 1, c is the speed of propagation of the tip, v is fluid velocity parallel with the bed and h is depth, measured perpendicular to the bed. Rewriting this in terms of the distance from the tip: $s = x_{\mathrm{tip}}\text{-}x$ and hence $\frac{\partial}{\partial s} = -\frac{\partial}{\partial x}$ we get

$$0 = -\tau + \varrho g h \frac{\partial h}{\partial s} + \frac{\partial}{\partial s} \int_{\mathrm{bed}}^{\mathrm{surface}} \varrho (v - c)^2 \mathrm{d}n \tag{2}$$

Here, the last term is additional to Whitham's (1955) quasi-steady balance between bed shear stress and pressure-force gradient because, we are dealing with a velocity distribution $v(s,n)$, while Whitham's and other hydraulics-style descriptions assume uniform velocity $v \equiv c$ in the tip. Since the propagation speed c of a tip, which propagates with constant form, is also the mean velocity in any vertical, the last term can also be seen as $\varrho \frac{\partial\, \mathrm{Var}(v)}{\partial s}$, ie, the derivative of fluid density times the velocity variance through a vertical. We shall express this term in terms of a momentum thickness δ_{m} defined by

$$\int_{\mathrm{bed}}^{\mathrm{surface}} \varrho (v - c)^2 \mathrm{d}n = \varrho c^2 \delta_{\mathrm{m}} \tag{3}$$

We shall see that, with the reasonable assumptions that δ_{m} scales in the same way as the boundary layer thickness with respect to the distance from the tip and/or the time since passage of the tip, namely

$$\delta_m \propto \delta \propto \sqrt{\nu_t t} \propto \sqrt{\nu_t s / c} \tag{4}$$

the new momentum equation (1) predicts observed tip surface shapes very successfully.

2.2. A simple self-similar velocity structure

It is instructive at this stage to go through the details for the simplest model: self-similar velocity profiles, eg,

$$v(s,n) = v_\infty \left[1 - \mathrm{e}^{-\frac{n}{\delta(s)}}\right] \tag{5}$$

where the asymptotic velocity v_∞ must be determined so that

$$\int_0^h v\, dn = ch \tag{6}$$

corresponding to the tip propagating with constant form at speed c. With this velocity structure, that gives

$$v_\infty = \frac{c}{1 - \frac{\delta}{h}\left[1 - \mathrm{e}^{-\frac{h}{\delta}}\right]} \tag{7}$$