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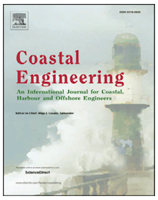

Sediment flux based model of instantaneous sediment transport due to pure velocity-skewed oscillatory sheet flow with boundary layer stream

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A R T I C L E I N F O

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A B S T R A C T

Velocity-skewed oscillatory sheet flow transport is studied by an approximate model for instantaneous sediment transport resulting from exponential profiles of sediment concentration and velocity over a mobile bed. The phase lag effect, and asymmetry in wave friction factor, oscillatory flow orbital amplitude, roughness height, bed shear stress and wave boundary layer thickness are modelled so that the net boundary layer stream of pure velocity-skewed oscillatory flow can be obtained. The proposed model is found consistent with the previous classical formulas, and qualitatively refines a series of exponents of velocity power function for the approximation of instantaneous sediment transport rate under different flow conditions. The sediment fluxes can be well predicted with accuracy as that of a two-phase numerical model. The net boundary layer stream and sediment transport rates are also well predicted, especially the negative net stream and rates with large phase residual. The phase residual is shown a useful but insufficient factor in the net sediment transport in pure velocity-skewed flows. In addition, the net boundary layer stream resulting from velocity-skewed wave boundary layer development difference between onshore and offshore flow durations is an essential factor as well.

1. Introduction

Velocity-skewed oscillatory flow is a result of wave propagation into a shallow water area. The sediment transport is complex, and the asymmetric development of the wave boundary layer is less straightforward under velocity-skewed oscillatory sheet flow with a sharp crest and a flat trough. Obtaining accurate knowledge of the corresponding sediment transport process is essential for fulfilling the coastal and ocean geomorphology study requirement. Consequently, many studies have been conducted on velocity-skewed flow sediment transport, which can be categorized into pure velocity-skewed flow (Dibajnia, 1991; Ribberink and Al-Salem, 1994; Ahmed and Sato, 2003; O'Donoghue and Wright, 2004a; b) and mixed flow with both velocity-skewness and acceleration-skewness (Ruessink et al., 2011; Dong et al., 2013; van der Z et al., 2015). Classical models of sediment concentration and velocity in boundary layer have been adequately expressed (Hurst, 1929; Rouse, 1937; Jonsson, 1966; Kajiura, 1968), and sediment size has also been discussed (Hassan and Ribberink, 2010; Kranenburg et al., 2014). The instantaneous sediment transport rate is calibrated as a power function of velocity with very different exponents (Sleath, 1978; Dick and Sleath, 1992).

The phase lag effect classifies the models for sediment transport in unsteady flow into a quasi-steady model and a semi-unsteady model (Silva et al., 2006; Hassan and Ribberink, 2010). Usually, three types of phase lag should be considered: (1) the phase residual (Dibajnia, 1991) which denotes the sediment entrained during the current half period, maintained in movement during deceleration stage, and transported after flow reversal; (2) the phase shift (O'Donoghue and Wright, 2004a) which denotes the time-delay of sediment movement in the sheet flow layer (erosion depth) to free stream velocity; and (3) the phase lead (Nielsen, 1992) which denotes the time-lead of bottom shear stress to free stream velocity. In the quasi-steady model, sediment transport is calculated immediately without the above three types of phase lag (Sleath, 1978; Nielsen, 1992, 2006; Ribberink, 1998). In the semi-unsteady models (Dohmen-Janssen et al., 2002; Watanabe and Sato, 2004; Silva et al., 2006; Dong et al., 2013; Lanckriet and Puleo, 2015), the phase residual and phase shift are usually seen while the phase lead is not considered. Net sediment transport rate is an important consequence of velocity skewness (Hassan and Ribberink, 2010), and it is mainly associated with the phase residual in the previous studies of pure velocity-skewed oscillatory sheet flows. Negative net sediment transport rate is attributed to

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large amount of sediments picked up during the onshore duration and transported during the offshore duration when the phase residual is very obvious for any condition with small sediment diameter, short period, or large velocity amplitude. Meanwhile, positive net sediment transport rate is observed to be proportional to velocity skewness and is generated when the phase residual effect is small. Moreover, Kranenburg et al. (2013) and Fuhrman et al. (2013) demonstrate that positive net boundary layer stream caused by progressive waves can re-reverse the net transport direction back to positive in conditions involving velocity-skewed waves and fine sediment.

Basic sediment transport mechanism in velocity-skewed oscillatory sheet flow is described through these widely-used models, but the prediction accuracy remains low, especially when more complex parameters are introduced. Most semi-unsteady models are half-period type (Dibajnia, 1991; Watanabe and Sato, 2004; Silva et al., 2006; Dong et al., 2013) that do not have any information about instantaneous sediment flux and transport rate. Many empirical models for instantaneous sediment transport exist (Sleath, 1978; Nielsen, 1992, 2006; Dohmen-Janssen et al., 2002; Ribberink, 1998), but they fail in pure velocity-skewed oscillatory sheet flow, because the phase residual is often excluded, and the velocity skewness generated net boundary layer stream is not considered. There exist several relevant complex factors needed in obtaining for instantaneous near-bed sediment flux and sediment transport rate. These factors, which include acceleration, phase lag, and asymmetric boundary layer development, are difficult to consider simultaneously in one model. Currently, an approximate model that considers the proposed complex factors in both velocity and concentration is helpful and important for obtaining a better description of instantaneous variation of sediment transport in pure velocity-skewed oscillatory sheet flow.

Thus, the current study attempts to use an approximate model for instantaneous sediment transport that considers the effects of acceleration, phase lag, and asymmetric boundary layer development to study the pure velocity-skewed oscillatory sheet flow transport. The profiles of velocity and concentration above a mobile bed in relation to erosion depth are integrated for instantaneous sediment flux and sediment transport rate. The net boundary layer stream is also studied and the power function exponent of velocity for instantaneous sediment transport rate is discussed.

2. Approximate model for instantaneous sediment transport

The approximate model for instantaneous sediment transport consists of velocity and concentration profiles, sediment flux and sediment transport rate. The variables and parameters are introduced in this section.

2.1. Velocity and concentration profiles

The pure velocity-skewed free stream velocity (**W**) is analytically approximated by Abreu et al. (2010) as follows:

$$\begin{aligned}\mathbf{W}(t) &= U_0 r \sum_{k=0}^{\infty} n^{-k} \exp\{i[(k+1)\sigma(t-t_0)-0.5k\pi]\} \\ &= U_0 r \frac{n\exp[i\sigma(t-t_0)]}{n-\exp\{i[\sigma(t-t_0)-0.5\pi]\}} \\ &= V(t)+iU(t)\end{aligned}, \qquad (1)$$

where the boldfaced notation represents a vector; U_0 is the amplitude of free stream velocity; r (=1-1/n^2) is a factor allowing velocity amplitude of Equation (1) to equal U_0; n (>1) is a wave form parameter; i is the imaginary unit; t is the time; $t_0 = -\sigma^{-1}\arcsin(n^{-1})$ forces $U(0) = 0$; σ (=2π/T) is the angular frequency; T is the period; V and U respectively are real and imaginary parts of **W**. Equation (1) can be written into the general form covering the Stokes wave as

$$\begin{aligned}\mathbf{W}(t) &= U_0 r \exp(0.5\pi i)\sum_{k=0}^{\infty} n^{-k} \exp\{i[(k+1)\sigma(t-t_0)-0.5(k+1)\pi]\} \\ &= U_0 r i \sum_{k=0}^{\infty} n^{-k} \exp\{i[(k+1)\sigma(t-t_0-0.25T)]\}\end{aligned}. \qquad (2)$$

Classical oscillatory flow under the 2nd order Stokes wave theory in O'Donoghue and Wright (2004a) corresponds to a velocity asymmetry parameter of $R = 0.5(1/n+1) = 0.625$. Thus, $n = 4$, $t_0 = -\sigma^{-1}\arcsin(0.25) = -0.04T$. By neglecting the terms of $k \geq 2$ in Equation (2) and forcing $r = 1$ to allow a velocity amplitude of U_0, the widely used velocity of the 2nd order Stokes wave is obtained as follows:

$$\mathbf{W}(t) = U_0 i\{\exp[i\sigma(t-0.21T)] + 0.25\exp[2i\sigma(t-0.21T)]\}. \qquad (3)$$

The imaginary part of Equation (3) is shown in Fig. 1, where subscripts c and t respectively denote the flow crest and flow trough; subscripts a and d respectively denote acceleration and deceleration; the positive and negative symbols respectively denote onshore and offshore directions; t_c, t_{rc}, t_t and t_{rt} are four special moments defined by $U(t_c)=U_c$, $U(t_t) = -U_t$ and $U(t_{rc}) = U(t_{rt}) = 0$; $0<t_c<t_{rc}<t_t<t_{rt}=T$. The velocity asymmetry parameter is also calculated as $R = U_c/(U_c + U_t) = 0.5U_c/U_0$.

Asymmetric variation of oscillatory, turbulent wave boundary layer is important in the generation of net boundary layer stream. An explicit approximation of boundary layer velocity is necessary for the present model. It can be approximated by a constant eddy viscosity and phase lead of 45° at a fixed bed surface, but the accuracy would not be exact enough. There are many models for velocity profiles of wave boundary layer starting from the classical model like Jonsson (1966) and Kajiura (1968). Kajiura's (1968) model has three layers of eddy viscosity, i.e., the inner and outer layers with constant eddy viscosity, and the overlap layer which is analogous to the logarithmic part of a steady boundary layer with linear variation of eddy viscosity and expected to exist for almost smooth beds. Later, Nielsen (1992) develops an explicit approximation covering a rough, turbulent, fixed bed, and further extends it to a mobile bed that the phase lead should be much smaller than 45° (Nielsen and Guard, 2010) as

$$\mathbf{W}_B(y,t) = U_0\exp(i\sigma t)\left\{1-\exp\left[-(1+\alpha i)\left(\frac{y+\Delta}{\delta}\right)^p\right]\right\}, \qquad (4)$$

where subscript B denotes the wave boundary layer; α is a phase lead parameter to adjust the phase lead at the immobile bed surface close to an actual turbulent flow; y is the vertical coordinate whose origin is at the initial undisturbed bed; Δ is the erosion depth; δ and p are boundary layer parameters.

Nielsen and Guard's (2010) Equation (4) consists of a symmetric free stream part [$U_0\exp(i\sigma t)$] and a boundary layer part {1-exp[-(1+αi)(y+Δ)$^p\delta^{-p}$]}. We try to apply Equation (4) for the engineering

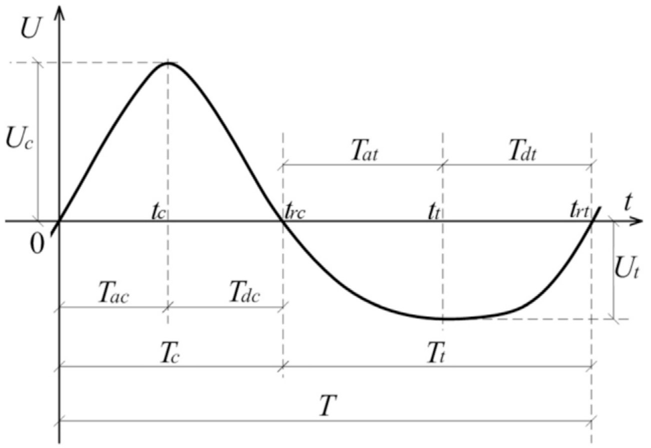

Fig. 1. Sketch of free stream velocity in pure velocity-skewed oscillatory flow ($n = 4$).