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Nonlinear wave crest distribution on a vertical breakwater

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ABSTRACT

The probability distribution of the nonlinear, up to the second order, crest height on a vertical wall is determined under the assumption of finite spectral bandwidth, finite water depth and long-crested waves. The distribution is derived by relying on the Quasi-Deterministic representation of the free surface elevation on the vertical wall. The theoretical results are compared against experimental data obtained by utilizing a compressive sensing algorithm for reconstructing the free surface elevation on the wall. The reconstruction is pursued by starting from recorded wave pressure time histories obtained by utilizing a sequence of pressure transducers located at various levels. The comparison demonstrates an excellent agreement between the proposed distribution and the experimental data, while, notably, the deviation of the crest height distribution from the Rayleigh one is considerable.

1. Introduction

The probability distribution of the crest heights in a sea state is a fundamental quantity for designing any coastal structure. The first investigation on the statistical properties of ocean waves was proposed by Longuet - Higgins (1952), who proved that linear crest heights are distributed according to the Rayleigh distribution in case of narrow-band spectra. Successively, Boccotti (1989, 2000, 2014) showed that the highest waves are distributed according to a Rayleigh distribution also for finite spectral bandwidths. In real seas, waves behave nonlinearly and this implies that their wave profile is modified strongly with respect to the linear case. Indeed, nonlinearity produces sharper and larger wave crests and flatter and smaller wave troughs.

Tayfun (1980, 1986a, 1986b, 1990) and Tung and Huang (1985) investigated the statistical properties of the nonlinear crest and trough wave amplitudes in an undisturbed wave field for the case of narrow-band spectra. A more general study was proposed by Arena and Fedele (2002) who introduced a bi-parametric family of non-linear stochastic processes representing the sea surface elevation and the wave pressure fluctuation both for progressive waves and for waves interacting with structures. Specifically, the exceedance probability of the absolute maximum and of the absolute minimum was derived for narrow-band processes, up to the second order. The case of finite spectral bandwidth in undisturbed wave field was developed theoretically and validated by both experimental data and numerical simulations by Forristall et al. (2000), Prevosto et al. (2000) and Fedele and Arena (2005). Tayfun (2006) investigated the statistics of nonlinear wave crests and wave-crest groups in deep and transitional water depths, using theoretical expressions describing the statistics of nonlinear wave crests and their groups in the form of a second-order transformation of well-known results on linear waves. Fedele and Tayfun (2009) presented a second-order model for weakly nonlinear waves and developed theoretical expressions for the expected shape of very large surface displacements. Arena and Guedes Soares (2009) developed analytical solutions of the nonlinear crest and trough amplitudes in bimodal sea states. Specifically, they extended the expression developed by Fedele and Arena (2005) by modifying the distribution parameters to account for sea states with double peaked spectra. Arena and Ascanelli (2010) derived the nonlinear crest heights distribution for three dimensional waves in a finite water depth. More recently, Zhang et al. (2015a) investigated the statistical properties of long-crested nonlinear waves measured in an offshore basin, in terms of surface elevation, wave crest and trough, and wave period and Wang (2018) proposed a transformed Rayleigh distribution of trough depth for stochastic ocean waves, using as transformation model a monotonic exponential function, calibrated such that the first three moments of the transformation model equal those of the real process.

Pelinovsky et al. (2008) derived a solution for the nonlinear shallow water equation of the wave field in front of a vertical wall and determined

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the crest height as a function of the incident wave height. Then, they used this solution to determine the exceedance probability of crest and trough amplitudes on the vertical wall. They demonstrated that freak wave events can occur at the breakwaters. In such conditions, nonlinearities are more relevant and high order models could be needed (Zhang et al., 2016).

This paper addresses the problem of determining the nonlinear wave crest amplitude distributions on a vertical wall at a finite water depth. Specifically, the crest height distribution is derived within the framework of the second order Stokes' wave theory in a reflected wave field via a solution based on the method discussed by Romolo and Arena (2008) and validated by means of experimental data recorded at Natural Ocean Engineering Laboratory (NOEL) of Reggio Calabria.

2. Theoretical distribution of nonlinear crest and trough amplitude on a vertical wall

The crest height distribution is derived by considering the Quasi-Deterministic representation of the free surface displacement in a reflected wave field at the seawall. This representation was given by Boccotti (2014) for a Gaussian sea, and by Romolo and Arena (2008) by approximating the free surface to the second order in a Stokes' expansion (a similar problem was addressed via a different approach by Sun and Zhang (2017)). In this context, if we consider a very high crest occurring at point y_0 at time t_0, the first and second order components of the free surface displacement are given by the equations

$$\overline{\eta}_{1R}(y_0, t_0+T) = \pm\frac{4h_0}{\sigma_R^2}\int_0^\infty S(\omega)\cos(\omega T)\cos^2(ky_0)d\omega, \tag{1}$$

and

$$\begin{aligned}\overline{\eta}_{2R}(y_0,t_0+T) = &-\frac{\Xi}{g}+\frac{2h_0^2}{\sigma_R^4}\int_0^\infty\int_0^\infty S(\omega_i)S(\omega_j)\cos(k_iy_0)\cos(k_jy_0)\cdot\\ &\left(\left\{A_{ij_1}^-\cos\left[(k_i-k_j)y_0\right]+A_{ij_2}^-\cos\left[(k_i+k_j)y_0\right]\right\}\cos\left[(\omega_i-\omega_j)T\right]+\right.\\ &\left.\left\{A_{ij_1}^+\cos\left[(k_i+k_j)y_0\right]+A_{ij_2}^+\cos\left[(k_i-k_j)y_0\right]\right\}\cos\left[(\omega_i+\omega_j)T\right]d\omega_jd\omega_i\right.\end{aligned} \tag{2}$$

where A_{ij1}^-, A_{ij2}^-, A_{ij1}^+, A_{ij2}^+ are the interaction kernels of the nonlinear free surface displacement, Ξ is a coefficient obtained by enforcing that the mean free surface displacement is zero in the time domain at any fixed point y_0 (see Appendix for their analytical formulae), $S(\omega)$ is the frequency spectrum of the incident free surface displacement, h_0 is the linear crest height which is assumed to be very large with respect to the mean wave crest amplitude (i.e. $h_0/\sigma_R \to\infty$) and σ_R^2 is the variance of free surface elevation in a reflected wave field to the first order in a Stokes' expansion.

From eq. (1) and eq. (2), if we assume that the nonlinear crest amplitude h_C occurs at $y_0=0$, $T=0$, it can be expressed as

$$\begin{aligned}h_C = h_0 + 2\frac{h_0^2}{\sigma_R^4}\int_0^\infty\int_0^\infty S(\omega_i)S(\omega_j)\Big\{&-2k_i\tanh(k_id)\delta_{ij}+\left(A_{ij_1}^-+A_{ij_2}^-\right)\\ &+\left(A_{ij_1}^++A_{ij_2}^+\right)\Big\}d\omega_jd\omega_i,\end{aligned} \tag{3}$$

where δ_{ij} is 1, for $\omega_i=\omega_j$, or 0 otherwise.

The Quasi Deterministic representation utilized in this paper is used for the free surface displacement in a Gaussian sea state in the vicinity of a very high (compared to the mean wave crest amplitude) wave crest. As a corollary of the underlying theory, Boccotti (2014), in agreement with Longuet - Higgins (1952), proved that in a Gaussian sea state, crest and trough amplitudes follow the Rayleigh distribution even for finite spectral bandwidths. Therefore, considering that the nonlinear second order crest height h_C is a quadratic function of the linear crest height h_0, its exceedance probability can be easily determined by means of the Rayleigh distribution of h_0.

According to Eq. (3), the dimensionless wave crest $\xi_{crest}=h_C/\sigma_\eta$, with σ_η being the standard deviation of the free surface displacement($\eta=\overline{\eta}_{1R}+\overline{\eta}_{2R}$), can be written as (Fedele and Arena, 2005),

$$\xi_{crest} = u\beta+\alpha(S)\beta u^2, \tag{4}$$

where $u=h_0/\sigma_\eta$ is the linear dimensionless wave crest, $\alpha(S)$ is a parameter representing the magnitude of the second order effects, and

$$\beta = \frac{\sigma_R}{\sigma_\eta}. \tag{5}$$

In Eq. (5) σ_R and σ_η are the standard deviations of the linear and the corrected up to second order free surface displacements, respectively.

Next, starting from Eq. (5), the analytical distribution $P(\xi_{crest}>\xi)$ is determined by obtaining the formal roots of the variable u from Eq. (4) and finding the conditions on u that satisfy the inequality $\xi_{crest}>\xi$. The following expression is obtained:

$$P(\xi_{crest}>\xi) = \exp\left[-\frac{1}{8\alpha^2}\left(1-\sqrt{1+\frac{4\alpha\xi}{\beta}}\right)^2\right]. \tag{6}$$

Clearly, the determination of the probability distribution (6) relies on the calculation of the parameters α and β. The former is estimated by utilizing the representation (3) in conjunction with the dimensionless amplitude (4) yielding

$$\begin{aligned}\alpha(S) = \frac{2}{\sigma_R^3}\Bigg[&-\int_0^\infty\int_0^\infty S(\omega_i)S(\omega_j)2k_i\tanh(k_id)\delta_{ij}d\omega_jd\omega_i\\ &+\int_0^\infty\int_0^\infty S(\omega_i)S(\omega_j)\left[\left(A_{ij_1}^-+A_{ij_2}^-\right)+\left(A_{ij_1}^++A_{ij_2}^+\right)\right]d\omega_jd\omega_i\Bigg].\end{aligned} \tag{7}$$

The latter is calculated by eq. (5). For this purpose, it is seen that, given the representations of the linear and of the nonlinear free surface displacements in a random reflected wave field at a given point y^* in the time domain,

$$\eta_{R1}(y^*,t) = 2\sum_{m=1}^M q_m\cos(k_my^*)\cos(\omega_mt+\varepsilon_m), \tag{8}$$

and

$$\begin{aligned}\eta_{R2}(y^*,t) = \frac{1}{2}\sum_{m=1}^M\sum_{n=1}^M q_mq_n\Big(&\left\{A_{mn_1}^-\cos\left[(k_m-k_n)y^*\right]+A_{mn_2}^-\cos\left[(k_m+k_n)y^*\right]\right\}\cos[(\omega_mt+\varepsilon_m)-(\omega_nt+\varepsilon_n)]+\\ &\left\{A_{mn_1}^+\cos\left[(k_m+k_n)y^*\right]+A_{mn_2}^+\cos\left[(k_m-k_n)y^*\right]\right\}\cos[(\omega_mt+\varepsilon_m)+(\omega_nt+\varepsilon_n)]+\\ &\sum_{m=1}^M q_m^2\left[\frac{k_m}{\sinh(2k_md)}-\frac{k_m}{\tanh(2k_md)}\cos(2k_my^*)\right],\end{aligned} \tag{9}$$