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# A process based shape equation for a static equilibrium beach planform

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ABSTRACT

Coastal erosion is a major problem for shorelines everywhere in the world. In order to mitigate beach stability problems it is necessary to determine the equilibrium shape of the beach planform. Current equilibrium planform shape equations are only valid under certain simplifications; for instance, they are unable to predict the effect of nearshore islands and/or rocky bottoms, as well as the effect of several diffraction points. The aim of this paper is to present a new process-based shape equation that is able to overcome those limitations and estimate the static equilibrium shoreline of complex bathymetry beaches. The equation is based on the hypothesis that a pocket beach gets its static equilibrium planform when the mean surf-zone longshore velocity averaged over a period of time is null ($\overline{V} = 0$) in every point along the beach. Based on this hypothesis, the direction of the shoreline that nulls the mean surf zone longshore velocity along the beach is evaluated. Therefore, the shape equation is based on the longshore current velocity formula. The new equilibrium shape equation is applied to two Spanish beaches, namely Milagro beach and Cala Millor beach. Milagro beach has a very smooth bathymetry and the parabolic shape equation developed by Hsu and Evans (1989) is able to predict the equilibrium shoreline. Cala Millor is more complex: it has some rock outcrops along the beach and the parabolic shape equation does not work. The equilibrium shape equation presented in this paper simulates the equilibrium shoreline of both beaches successfully with a high $R^2$ (around 0.96) between the modelled shoreline and the real shoreline in both case studies.

## 1. Introduction

Beaches are important recreational areas, attracting an ever increasing number of users worldwide (Vaz et al., 2009). Their importance as an economic driver is significant for many countries (Houston, 1995). However, coastal areas around the world are threatened by beach erosion (Alexandrakis et al., 2015). This sediment supply depletion can be caused by many different processes, such as alterations in longshore drift due to man-made structures, sea level rise, dredging and cross-shore processes during storms. The first step to mitigate beach erosion in addition to coastal management is to determine the equilibrium shape of the beach planform in order to check and test the stability of the beach and provide solutions for erosion problems. Furthermore, the equilibrium beach planform is used as an engineering tool for the design of nourishment projects (González et al., 2010) and the creation of new beaches (Elshinnawy et al., 2017; Hsu et al., 2010).

Initially, equilibrium beach planform equations were studied by geographers and geologists (Halligan, 1906; Yasso, 1965; King, 1972; Carter, 1988). It was in the 60s that engineers started paying attention to the stability of embayments (Silvester, 1960; Hsu and Evans, 1989; Silvester and Hsu, 1993). For this purpose, the most widely used formula is the one proposed by (Hsu et al., 1987), which considers that bays in static equilibrium reach a parabolic shape. Hsu and Evans (1989) modified the parabolic shape equation by adding polar coordinates for the curved section of the beach. Since then, different authors have suggested improvements (Tan and Chiew, 1994; González and Medina, 2001) to the expression proposed by Hsu and Evans (1989) and methodologies for its application. However, current equilibrium shape equations are not derived directly from the acting physical processes that developed the shape; rather, they are empirical equations.

Consequently, these empirical shape equations are valid taking into account certain simplifications. Therefore, they are unable to predict the effect of nearshore islands and/or rocky bottoms, as well as the effect of several diffraction points (González, 1995).

One of the main hypotheses that lies behind these shape equations is that a beach reaches its static equilibrium planform when the littoral drift is almost nonexistent and external sediment is not required to maintain its longterm stability (Hsu et al., 2010). This hypothesis is in agreement with the experimental results obtained by (Ho, 1971) and (Yamashita and Tsuchiya, 1992), who concluded that a beach reaches a static equilibrium shape when the mean surf-zone velocity averaged over a period of time is negligible ($\overline{V} = 0$).

The current theories on the formation of longshore currents (Uchiyama et al., 2010; Kumar et al., 2011) were first stated in the papers written by (Bowen, 1969; Longuet-Higgins, 1970; Thornton, 1970). They employed the concept of radiation stress to describe the flux of the momentum associated with the incoming waves. In order to obtain simple equations, some assumptions are normally made, like neglecting the

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turbulence term, etc. Most of the prevalent longshore current formulas assume that the two main mechanisms for generating currents are wave obliquity and wave height gradient, so if neither of them exists there is no longshore current at all.

As for the longshore current profile, Longuet-Higgins (1970) demonstrated that the velocity profile across the surf zone reaches a maximum at the breaker line and decreases linearly to zero at the shoreline. The velocity is also zero beyond the breaker zone leading to an abrupt discontinuity at the breaker line (Bowen, 1969; Longuet-Higgins, 1970; Thornton, 1970). included horizontal mixing in their analyses and, consequently, obtained a more realistic longshore velocity profile without that abrupt discontinuity. It is generally envisaged that horizontal eddies, originating primarily from the water-breaking processes, will produce a horizontal transfer of momentum. Therefore, the horizontal mixing is important when characterising cross-shore velocity profiles. Other researchers (Nadaoka et al., 1992; http://www.sciencedirect.com/science/article/pii/S0378383900000053 - BIB22, Kuriyama, 1994, Osiecki and Dally, 1996) included the roller effect into predictive models for longshore current. However, on the basis of simulation results with and without the roller effect (Reniers and Battjes, 1997), concluded that the surface roller has a small contribution.

The objective of this paper is to present a new equilibrium (long-term scale) beach planform shape equation, since current static equilibrium beach planform shape equations cannot be used on beaches that are affected by islands or slabs of rock. The new planform shape equation is based on the hypothesis that the mean surf-zone velocity averaged over a period of time is negligible, also taking into account the hydrodynamic processes and the fact that it can be applied in beaches with a complex geometry. This paper is organised as follows. The hypothesis, the governor equations and the methodology to apply the equilibrium planform shape equation are presented in section 2. In section 3, two different case studies are described. Finally, in section 4 and 5, the discussion and the conclusions are presented respectively.

## 2. Shape equation description

In this section, the main hypothesis and assumptions that govern the beach planform shape equation are described. Based on those assumptions, the longshore velocity equation is obtained. Afterward, the static equilibrium planform shape equation is developed. Finally, the methodology is described. First the sea state that represents the whole time series needs to be obtained and then, based on that sea state, the planform shape has to be calculate by applying the new equilibrium planform equation.

### 2.1. Hypothesis and assumptions

The static equilibrium planform shape equation developed in this study is based on the hypothesis that a pocket beach is in static equilibrium when the mean surf-zone velocity averaged over a period of time is null ($\overline{V} = 0$) in every point along the beach (Ho, 1971; Yamashita and Tsuchiya, 1992; González and Medina, 2001). Based on this hypothesis, the direction of the shoreline that nulls the mean surf zone longshore velocity along the beach is evaluated.

The longshore current formula has been developed following the concept of radiation stress (Longuet-Higgins and Stewart, 1962).

The following hypotheses are assumed (Longuet-Higgins, 1970): (1) small wave incidence angle at the breaking point, (2) constant bed slope (tanβ), (3) mean-flow convective terms are neglected, (4) local variation of the mean values are neglected and (5) the breaking criteria, Hb = γD, is used, where Hb is the significant wave height before the breaking, γ is 0.55 (Riedel and Byrne, 1986) and D refers to the total water depth (see Fig. 1).

The y-axis and x-axis of the reference coordinate system lie parallel to the beach and normal to the shoreline, respectively.

The conservation of the horizontal momentum equation is obtained by integrating the momentum equation over depth. The balance of total momentum per unit area can be expressed as (Mei, 1989):

$$\frac{\partial}{\partial \mathrm{x}}\left(\mathrm{S_{yx}} + \mathrm{S'_{yx}}\right) + \frac{\partial}{\partial \mathrm{y}}\left(\mathrm{S_{yy}} + \mathrm{S'_{yy}}\right) = \mathrm{T_y} + \mathrm{R_y} \tag{1}$$

where

$$T_y = -\rho g D \gamma K_1 \frac{\partial H_b}{\partial y},$$

$$R_y = \frac{-\rho C_f}{\pi} \gamma \sqrt{gD} \cdot V$$

and where $\rho$ is the sea water density (1035 Kg/m3), $g$ the gravitational acceleration (9.8 m/s2), $H$ the wave height, $Cf$ the drag coefficient (0.01 (USACE, 1984)), $V$ the longshore current velocity, $h$ the water depth, $x$ the cross-shore distance (m), $\overline{\eta}$ the wave set-up and $K1 = \frac{5\gamma}{2(3\gamma^2+8)}$. The subscript $b$ means that the parameter is calculated at the breaking point.

In equation (1), $S_{yx}$ and $S_{yy}$ are the radiation stress tensors and $S'_{yx}$ and $S'_{yy}$ are the Reynolds stress tensors which are defined in (Longuet-Higgins and Stewart, 1962). $T_y$ represents the net horizontal force per unit area due to the slope of the free water surface and $R_y$ is the mean averaged shear stress which must be included in any realistic treatment of the surf zone where dissipative effects occur.

By expanding each term of equation (1), the longshore velocity is obtained:

$$V = \theta_{comp} + \partial_{comp} + T_{comp}, \tag{2}$$

where

$$\theta_{comp} = \frac{5\pi}{16C_f} g \tan^2 \beta \gamma \frac{1}{\sqrt{gD}} x \sin\boldsymbol{\theta_b} \tag{3}$$

$$\partial_{comp} = -\frac{\pi}{C_f}(gD)^{½} K_1 \frac{\partial \boldsymbol{H_b}}{\partial \boldsymbol{y}} \tag{4}$$

$$T_{comp} = T1_{comp} + T2_{comp} \tag{5}$$

$$T1_{comp} = -\frac{10\pi^2 \varepsilon (\tan\beta)^{½}}{16 c_f^2 \gamma h_b^{½}} K_1 \gamma \frac{1}{x^{½}} \left(\frac{\partial H_b}{\partial y}\right)^2 \tag{6}$$

$$T2_{comp} = -\frac{10\pi^2 \varepsilon}{16 c_f^2 \gamma h_b^{½}} \tan\beta D \left(\frac{\partial^2 H_b}{\partial y^2}\right) \tag{7}$$

and where $\theta_b$ is the angle between the shoreline and the wave crest, β the beach profile slope and ε the eddy viscosity.

The first term ($\theta_{comp}$) in the longshore velocity formula (equation (2)) refers to the obliquity of the wave, the second term ($\partial_{comp}$) to the wave height gradient and the third term ($T_{comp}$) to the turbulence term which consists of two different parts ($T1_{comp}$ and $T2_{comp}$).

The turbulence term is commonly neglected when calculating the longshore current. However, in the equilibrium planform equation the turbulence generated by the wave breaking affects the accuracy of the estimated planform shoreline. Significance differences exist depending on whether the turbulence term is added or neglected (Gainza et al., 2017). In the latter case, the final shoreline does not curve as much as it should do, but instead it remains straight along the beach and forms abruptly into a curve close to the diffraction point. In real beaches, in the contrary, the shoreline is smooth and starts curving gently far from the diffraction point. The inclusion of the turbulence term effectively couples together the adjacent elemental water columns, resulting in diffusion in a direction parallel to the shore. This lateral diffusion of momentum