Coastal Engineering 133 (2018) 13–25

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

Hysteresis in the evolution of beach profile parameters under sequences of wave climates - Part 2; Modelling

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ARTICLE INFO

Keywords:
Equilibrium beach state models
Sediment transport
Beach erosion
Beach accretion
Morphological hysteresis

ABSTRACT

Disequilibrium-type models for two beach profile parameters, *P*, the shoreline position and net bulk sediment transport, are developed for laboratory experiments that demonstrate morphological hysteresis in the evolution to equilibrium of beach profiles under sequences of different wave climates. The model principle follows the classical disequilibrium approach but with non-monotonic relationships between the forcing and the chosen beach profile parameter at equilibrium, P_{eq}, previously verified and presented in part 1 of this work (Baldock et al., 2017). Two such relationships are required to model beach profile evolution that exhibits morphological hysteresis. The model coefficients are derived for monochromatic and random wave experiments and subsequently used to model data obtained from cyclic erosive and accretive wave conditions of shorter durations, alternating through multiple cycles. In these conditions equilibrium conditions were not reached and hysteresis does not occur. The model is used to investigate the morphological feedback between the outer and inner bars and the resulting behaviour of the bulk transport, and the relative depth over the bar crest is shown to be an attractor in this case. The model coefficients and morphological time-scales derived from the cyclic experiments are very similar to those derived from the equilibrium experiments for the bulk transport. Normalised mean square model errors range from 1% to 20% when applied to independent data. The data from the cyclic wave conditions can be inverted to derive the conditions expected at equilibrium, which match those observed, indicating a robust model relationship between the forcing and P_{eq}. The relationship between the forcing and P_{eq} can also be determined directly from the cyclic experiments. This approach may be more robust than determining the relationship from periods where *P* is stationary since, in a time-series of *P* versus the forcing, stationary points can occur due to changes in wave conditions, in addition to the instances when $P=P_{eq}$.

1. Introduction

Models for the evolution of a beach profile parameter, such as the shoreline position, toward an equilibrium condition are commonly used to describe and predict beach behaviour (Sunamura, 1983; Wright et al., 1985; Kriebel and Dean, 1985; Miller and Dean, 2004). Further refinements and extensive testing of shoreline evolution models have been documented by Yates et al. (2009, 2011), Davidson et al. (2010, 2013) and Splinter et al. (2014). In all these models, there is no morphological hysteresis, in that after a beach reaches equilibrium following an earlier increase in the forcing, a subsequent reduction of the forcing reverses the motion of the shoreline, albeit at a different rate. However, in part 1 of this work (Baldock et al., 2017), laboratory experiments demonstrated a strong hysteresis in beach profile evolution during sequences of wave conditions with increasing and then decreasing energy. Notably, after equilibrium conditions were reached during strongly erosive wave conditions, erosion continued after a reduction in forcing. Consequently, a single relationship between the forcing and the equilibrium condition (*e.g.* Yates et al., 2009) is insufficient to model the beach behaviour, and two relationships are needed.

Baldock et al. (2017) proposed that this can be reconciled with the classical idea of a single beach state at equilibrium for any given forcing, as in the classical Wright et al. (1985) model, through the concept of a subsequent alternate active beach state. Thus, a reduction in wave height over the existing morphology (the antecedent equilibrium beach state) may result in a more reflective beach profile, which then continues to erode as it evolves to a more dissipative equilibrium state for those new wave conditions.

In the laboratory experiments, the physical mechanism for continued erosion following a reduction in wave height was the stranding and

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https://doi.org/10.1016/j.coastaleng.2017.12.001
Received 14 December 2016; Received in revised form 3 October 2017; Accepted 2 December 2017

subsequent destruction of the original breaker bar, with the simultaneous generation and offshore propagation of a new breaker bar originating in the inner surf zone. Data from large-scale experiments show similar behaviour (Eichentopf et al., 2017). Conversely, shorter duration experiments with cycles of erosion and accretion, where equilibrium was not reached, showed no such hysteresis.

The present paper attempts to model firstly the hysteresis in the behaviour of two beach profile parameters, the shoreline position and the net bulk sediment transport, observed in the laboratory experiments, which provides model coefficients analogous to those derived from time-series of shoreline position observed in the field (*e.g.* Yates et al., 2009; Davidson et al., 2013; Splinter et al., 2014). Secondly, a conceptual model, which accounts for feedback between the net bulk sediment transport and the bar position or relative depth over the bar is presented to better describe episodes when morphological hysteresis occurs. Finally, the evolution model is tested against data from experiments with cyclical erosive and accretive wave conditions of shorter duration, both with the coefficients derived from the equilibrium experiments and a new set of coefficients, the latter optimized for the cyclical experiments. Given the thorough discussion of previous models and the laboratory experiments in part 1, the paper commences with an outline of the modelling approach (section 2), followed by a brief summary of the experimental conditions (section 3). Section 4 first presents the methodology to optimise the model coefficients for the equilibrium beach profile conditions and outlines the resulting model errors. Subsequently, the modelling of the evolution of the beach state parameters and the morphological hysteresis sequences is presented. Finally, the application and optimisation of the model to the cyclic wave conditions is discussed, together with an analysis of the differences in model performance and the variation of the coefficients. Final conclusions follow in section 5.

2. Modelling

2.1. Previous formulations

The model uses the widely adopted classical disequilibrium concept introduced by Sunamura (1983) and Wright et al. (1985), and refined by many others since. For a given beach profile parameter, P, (*e.g.* the shoreline location) there are typically two models for the rate of change of that parameter with time. Firstly, dP/dt is a direct function of P via the present disequilibrium ΔP between the current beach state P and an equilibrium state P_{eq}, the latter related to a given wave condition (Plant et al., 1999; Miller and Dean, 2004, 2006). Secondly, dP/dt is an indirect function of P via $\Delta\Omega$, the disequilibrium of the conventional beach state parameter, Ω, or the dimensionless fall velocity, $H/\omega T$ (Gourlay, 1968; Dean, 1973) with respect to an equilibrium beach state Ω_{eq} (Wright et al., 1985; Davidson et al., 2010, 2013; Splinter et al., 2013, 2014).

For the first configuration, the classical equilibrium model simply reads:

$$\frac{dP}{dt} = \frac{P_{eq} - P}{\tau} = \frac{\Delta P}{\tau} \tag{1}$$

where τ is the morphological timescale. Stationary points in $P(t)$ theoretically correspond to $P=P_{eq}$. Taking $P(0) = P_0$, the general solution $P(t)$ reads:

$$P(t) = P_{eq} + \left(P_0 - P_{eq}\right)e^{-\frac{t}{\tau}} \tag{2}$$

For the second configuration, Wright et al. (1985) suggested:

$$\frac{dP}{dt} = b + a\frac{\Omega - \Omega_{eq}}{\tau} = b + \frac{a}{\tau}\,\Delta\Omega \tag{3}$$

$P(t)$ is estimated with:

$$P(t) = c + bt + a\int_0^t \frac{\Delta\Omega}{\tau} \tag{4}$$

The linear trend $c + bt$ accounts for external sand supply/sinks through longshore transport or beach nourishment/dredging. In both cases, any linear trend in the data is traditionally removed from the data beforehand. Davidson et al. (2013) and Splinter et al. (2014) proposed further improvements for the model. Firstly, they suggested to use the wave power P_W ($P_w = EC_g = \frac{\rho g^2}{32\,\pi}\; H_s^2\, T_p$) to the power ½ instead of Ω. The power p = ½ conserves $\tau \propto H_s^{-1}$ as in Yates et al. (2009). Secondly, Ω_{eq} accounts for the preceding history in wave conditions through a memory decay function (or response factor), which governs the weighting of prior antecedent wave conditions.

Yates et al. (2009) adopted a hybrid approach, defining the rate of change as a function of the disequilibrium in wave energy ΔE and the instantaneous energy E (to the power ½):

$$\frac{dP}{dt} = C^{\pm}\,E^{0.5}\left(E - E_{eq}\right) = C^{\pm}\,E^{0.5}\Delta E \tag{5}$$

with $C^{\pm}$ for accretive or erosive conditions. Yates et al. (2009) defined a linear relation between a given shoreline position P and an equilibrium wave energy E_{eq} that causes no change

$$E_{eq} = aP + b \leftrightarrow P = \frac{E_{eq} - b}{a} \tag{6}$$

Reciprocally, each given wave energy E allows the existence of one (and only one in the linear case) equilibrium shoreline position P_{eq}:

$$E = aP_{eq} + b \leftrightarrow P_{eq} = \frac{E - b}{a} \tag{7}$$

dP/dt can be rewritten as:

$$\frac{dP}{dt} = C^{\pm}\,E^{0.5}\left(E - E_{eq}\right) = C^{\pm}\,E^{0.5}\left(-\,a\,\left(P - P_{eq}\right)\right) = -aC^{\pm}\,E^{0.5}\left(P - P_{eq}\right) \tag{8}$$

which is equivalent to the first configuration (eq. (1))

$$\frac{dP}{dt} = -\frac{P - P_{eq}}{\tau} = \frac{\Delta P}{\tau}$$

with $\tau = \frac{1}{aC^{\pm}\,E^{0.5}} = \frac{E^{-0.5}}{aC^{\pm}}$. The model contains 4 free parameters; the linear coefficients a and b, and the erosion/accretion related coefficients C^+ and C^-. Yates et al. (2009) used simulated annealing (*e.g.* Kirkpatrick et al., 1983) and a surrogate management framework (*e.g.* Booker et al., 1999) to find the best estimate of the free parameters (considering root mean square error, RMSE, minimization). Doria et al. (2016) proposed further improvements for the model with the implementation of limiters for shoreline recession in the presence of hard structures and ensuring that the equilibrium wave energy E_{eq} does not turn negative for significantly accreted profiles. Other formulations for the wave energy/shoreline relation (cf. equations (6) and (7)) were tested in order to reduce the number of free parameters. Both linear and nonlinear monotonic formulations showed similar performance for the shoreline position prediction.

The morphological time scale τ controls the rate of convergence toward the equilibrium state and clearly depends on the incident wave conditions. Miller and Dean (2004, 2006) and Yates et al. (2009) proposed and tested several alternative parametrizations for τ depending on, for instance, the breaking wave height H_b, the radiation stress S_{xx}, the offshore wave steepness H_0/L_0, the wave energy E, or the dimensionless fall velocity, Ω, to some power $p = 1, 2$. The latter formulation is analogous to the Wright et al. (1985) choice of disequilibrium in Ω. Sensitivity analysis showed similar model performance while interchanging those different parametrizations. This is to be expected if the different free parameters are implicitly correlated or optimized. An optimisation algorithm (*e.g.* simulated annealing, least-square estimator) is a minimization strategy, *i.e.* any combination of free parameters shows a best fit and a constraining physical range may require specification for realistic