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Transformed Rayleigh distribution of trough depths for stochastic ocean waves

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ABSTRACT

This paper deals with the calculation of wave trough depths exceedance probabilities for stochastic ocean waves, and a Transformed Rayleigh method is proposed for carrying out the calculation. In the proposed Transformed Rayleigh method, the transformation model is chosen to be a monotonic exponential function, calibrated such that the first three moments of the transformed model match the moments of the true process. The detailed mathematical procedures and formulas of the proposed Transformed Rayleigh method are given in this article, and the second order Stokes wave model has been integrated into this Transformed Rayleigh method. The proposed new method has been applied for calculating the wave trough depths exceedance probabilities of two sea states, one with the surface elevation data measured at the coast of Yura in the Japan Sea, and another one with the surface elevation data measured at the North Sea. It is demonstrated that the proposed new method can offer better predictions than those from using the theoretical Rayleigh wave trough depths distribution model. The calculation results from using the proposed new method are further compared with those obtained from using the Arhan and Plaisted nonlinear distribution model and the Toffoli et al. wave trough depths distribution model, and the accuracy of the new method has been once again substantiated. The research findings gained from this study demonstrate that the proposed Transformed Rayleigh method can be readily utilized in the process of designing various kinds of ocean engineering structures.

1. Introduction

The description of wave trough depths and their associated exceedance probabilities represents a key input for the design and risk assessment of various kinds of ocean engineering structures. When designing a Mobile Offshore Drilling Unit (MODU) such as a semi-submersible platform, it is necessary to define the maximal wave trough depth because the underwater cross-bars of the platform must not be exposed to the air, but at the same time should be sufficiently close to the water surface. The wave trough depths distribution is also very important in the process of design and risk assessment of a tension-leg platform (or a tension-leg platform floating wind turbine) because it is used for the calculation of the tether loads acting on the platform (Toffoli et al., 2008).

It is generally regarded that the wave trough (or crest) distributions obey the Rayleigh probability law in an ideal Gaussian narrow band random sea ((Romolo and Arena, 2015; Petrova and Soares, 2014; Latheef and Swan, 2013; Longuet-Higgins, 1952)). In the ideal Gaussian sea model the individual sine wave trains superimpose linearly (add) without interaction, and therefore, the model is also called the linear sea model. However, waves in the real world are nonlinear. Real waves show a small but easily noticed departure from a Gaussian surface. The crests are higher and sharper than expected from a summation of sinusoidal waves with random phase, and the troughs are shallower and smaller (Forristall, 2000). Consequently, the probability density function of the water surface in a real ocean tends to deviate from the Gaussian distribution: the larger crest heights are underestimated and the shallower trough amplitudes are overestimated by the Rayleigh's distribution. Therefore, the application of the Rayleigh distribution to the wave troughs (or crests) will become invalid, and other more suitable methods should be applied to predict the distribution of wave troughs (or crests) for the nonlinear random model of the sea elevation.

In the existing literature, the nonlinear wave trough distribution model has been very rare. Arhan and Plaisted (1981) and Toffoli et al. (Toffoli et al., 2008) produced wave trough probability distribution from

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the nonlinear Stokes wave model. However, Wang and Xia (2012) have shown that such kinds of theoretical and/or empirical models will sometimes predict wave characteristic distributions that differ considerably from the true ones. Fedele and Arena (2005) has also made a great effort in the attempt to model the effects of non-linearity that are observed in the real ocean field and developed a nonlinear wave trough distribution model. Although these authors have found important results which are in very good agreement with measured wave data, they stated in their paper (Fedele and Arena, 2005) that their wave trough distribution model is only applicable when the nonlinear effects are weak. In view of the above mentioned facts, the author of this article has proposed a Transformed Rayleigh method in order to calculate the wave trough distributions of irregular Stokes waves more accurately and efficiently. In this proposed Transformed Rayleigh method, the transformation model is chosen to be a monotonic exponential function, calibrated such that the first three moments of the transformed model match the moments of the true process. Information about the moments of the process can be obtained by resorting to theoretical models. In this article, the proposed new method will be applied to predict the wave trough exceedance probabilities of a sea state with the surface elevation data measured at the coast of Yura in the Japan Sea and of another sea state observation in the North Sea. Meanwhile, the wave trough exceedance probabilities of these sea states obtained from using the conventional Rayleigh distribution model and from using the Arhan and Plaisted model and Toffoli et al. model will also be included for comparison purpose. The calculation results will be analyzed, and some findings valuable for engineering design will be pointed out.

This paper begins in Section 2 by introducing the knowledge of the irregular Stokes waves. It continues in Section 3 by elucidating the theoretical background of the proposed Transformed Rayleigh method. In Section 4 the calculation examples and discussions will be provided, with concluding remarks summarized in Section 5.

2. The irregular Stokes waves

For an ideal linear Gaussian sea, the probability distribution of the wave troughs can be calculated according to the following Rayleigh law:

$$F(A_t < h_t) = 1 - \exp\left[-8\left(\frac{h_t}{H_s}\right)^2\right] \tag{1}$$

where h_t is the height of the wave trough, H_S is the significant wave height, and $F(\ )$ denotes a probability distribution function. In the Gaussian sea model the individual sine wave trains superimpose linearly (add) without interaction. Therefore, the model is also called the linear sea model. However, it is known that when the sea states become too severe or as the water depths decrease, the non-linearities of sea waves become more and more relevant, and the ocean surface elevation process deviates significantly from the Gaussian assumption.

For nonlinear Stokes waves, the non-Gaussian effects should be considered when predicting the wave trough distributions. Obviously, the Rayleigh distribution which is good for predicting the trough depths of linear Gaussian waves will become invalid for calculating the wave trough distributions of nonlinear Stokes waves. Arhan and Plaisted (1981) produced a wave trough probability distribution from the nonlinear Stokes wave model, which is given by:

$$F(A_t < h_t) = 1 - \exp\left[-\frac{8}{R_*^2}\left(\sqrt{1-\frac{2R_* h_t}{H_s}} - 1\right)^2\right] \tag{2}$$

where the wave effective steepness R_* is given by:

$$R_* = kH_s f_2(kd) = kH_s\left(\frac{\cosh kd(2+\cosh 2\,kd)}{2\sinh^3 kd} - \frac{1}{\sinh 2\,kd}\right) \tag{3}$$

where k is the wave number and d is the water depth.

Toffoli et al. (2008) proposed another model for the wave trough exceedance probability as follows:

$$P(A_t > h_t) = \exp\left[-\frac{8}{H_S^2 k^2}\left(\sqrt{1-2kh_t} - 1\right)^2\right] \tag{4}$$

where k is the wave number, and H_s is taken to be four times the standard deviation of the measured surface elevation.

However, Wang and Xia (Fedele and Arena, 2005) have shown that such kinds of theoretical and/or empirical models will sometimes predict wave characteristic distributions that differ considerably from the true ones (For detailed information regarding the differences between empirical distributions and the true ones, the readers should refer to Section 4.3 and Fig. 15 in Wang and Xia (Fedele and Arena, 2005)). Therefore, the author of this article has proposed a Transformed Rayleigh method in order to calculate the wave trough distributions of irregular Stokes waves more accurately and efficiently. In section 3 of this article the theoretical background of this Transformed Rayleigh method will be elucidated.

3. The Transformed Rayleigh method

It has been shown in Wang (2016) that the wave surface elevations $\eta(x, t)$ for the irregular Stokes waves at a specific reference location (say $x = 0$) can be written in the following matrix notation:

$$\eta(t) = \mathbf{s}^T\mathbf{x} + \mathbf{x}^T[Q+R]\mathbf{x} + \mathbf{y}^T[Q-R]\mathbf{y} \tag{5}$$

Please note that a similar expression for the wave surface elevation as equation (5) first appeared in Langley (1987). However, Langley (1987) solely deals with the distribution of ***wave surface elevation***, which is an entirely different physical quantity from a ***wave height*** or a ***wave trough***. Furthermore, the theory behind the proposed Transformed Rayleigh method in this paper is entirely different from the theory in Langley (1987).

In equation (5) Q and R are real symmetric matrix whose nmth components are $s_n s_m q_{mn}$ and $s_n s_m r_{mn}$ respectively, and q_{mn} and r_{mn} are quadratic transfer functions relating to the wave frequency range of a specific sea spectrum $S_{\eta\eta}(\omega)$. **s**, **x** and **y** are vectors whose nth components are s_n, x_n and y_n respectively and $s_n = \sqrt{S_{\eta\eta}(\omega_m)d\omega}$. x_m and y_m are Gaussian random variables at any fixed time t and have the following properties:

$$E[x^2{}_m] = E[y^2{}_m] = 1;\ E[x_m y_n] = 0 \tag{6}$$

$$E[x_m x_n] = E[y_m y_n] = 0; \quad m \neq n \tag{7}$$

By performing an eigenvalue decomposition, equation (5) becomes:

$$\eta(t) = \mathbf{s}^T\mathbf{x} + \mathbf{x}^T P_1^T \Lambda_1 P\mathbf{x} + \mathbf{y}^T P_2^T \Lambda_2 P_2\mathbf{y} \tag{8}$$

where Λ_i is a diagonal matrix with the eigenvalues in the respective diagonal and P_i contains the corresponding eigenvectors per row. Introducing a new set of Gaussian random variables Z_j, such that:

$$\mathbf{Z} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} \tag{9}$$

we can write the stochastic process $\eta(t)$ as (this is called the Kac-Siegert solution):

$$\eta(t) = \sum_{j=1}^{2N} \beta_j Z_j + \lambda_j Z_j^2 \tag{10}$$

where Z_j are independent Gaussian processes with unit variance, and β_j and λ_j are coefficients computed based on the information provided by the sea spectrum $S_{\eta\eta}(\omega)$, which is chosen for a given sea state.