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Experimental investigation of nonlinear regular wave transformation over a submerged step: Harmonic generation and wave height modulation

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ABSTRACT

In this study, a series of flume experiments were conducted to investigate the harmonic generation and wave height modulation (i.e., recursion) of nonlinear waves passing over a submerged step. A total of 35 experimental conditions were considered, including several different incident amplitudes/periods and bottom configurations, i.e., four step widths and three step heights (or three submergence depths). The waveforms across a 7.8-m-long segment were recorded over a period of 21.8 s using the non-intrusive imaging system. The measured surface elevations were further analyzed to reveal the amplitude variations of harmonic frequencies based on the 2D Fast Fourier and 1D Morlet wavelet transforms. A set of parameters was proposed to depict the generation of second-harmonic waves at the step crest. Overall, it was found that the most influencing factors for harmonic generation are the Ursell number of incident waves and relative height of the obstacle while the relative step width yields an insignificant impact. The wave height modulation (i.e., recursion) of second harmonic is affected mainly by the wavelengths of its free and bound components, which are accurately estimated from the Morlet wavelet transform analysis and third-order Stokes dispersion relation. The amplitude of primary wave exhibits a different recursion pattern due to the generation of the third, forth, and higher harmonics. Based on the concept of average energy flux, the recursion phenomenon of harmonics is further clarified.

1. Introduction

Nonlinear wave propagation over a submerged obstacle involving complicated harmonic generation processes is an important research topic in coastal engineering (Johnson et al., 1951). Particularly, the incident (or transmitted) wave energy can be dissipated and re-distributed by the well-designed submerged breakwaters, effectively reducing the potential hydrodynamic impacts (or forces) for environmentally-friendly coastal protection and hazard mitigation (e.g., Hughes and Fowler, 1995; Losada et al., 1997; Vilchez et al., 2016). Over the last few decades, a lot of theoretical (Mei and Black, 1969; Mei and Unluata, 1972; Massel, 1983), experimental (Dattatri et al., 1978; Beji and Battjes, 1993; Ting et al., 2004; Li and Ting, 2012), and numerical (Ohyama and Nadaoka, 1994; Christou et al., 2008; Young et al., 2009; Wu et al., 2010) efforts have been devoted to such an interesting wave problem for deeper physical insights, e.g., the generation criterion of super harmonics, characteristics of energy distribution, and variations of recursion length (beat length). When a wave passes from the deep-water region onto the submerged obstacle, the increased nonlinearity in the shallow-water area enhances the transferring of energy from the primary wave for higher harmonic generation. Each harmonic frequency (ω_n) consists of the bound (phase-locked) and free wave components which present different wave numbers (i.e., k_b and k_f) and traveling speeds (i.e., ω_n/k_b and ω_n/k_f). Accordingly, in response to the inherent phase mismatch, the spatial distribution of harmonic amplitude reveals an evident recursion behavior (Massel, 1983; Grue, 1992; Mei and Liu, 1993; Madsen and Sorensen, 1993).

The generation criterion, magnitude, and recurrence of higher harmonics (at the shallow-water region above the submerged obstacle) are influenced by the incident conditions (Ohyama and Nadaoka, 1994), i.e., wave height ($H_0 = 2a_0$) and period (T_0). For instance, the energy of the primary wave starts to shift toward the high frequency components while the ratio of the nonlinear parameter ($\delta = a_0/h_s$, where h_s represents the depth at the shallow-water area) to the dispersion parameter ($\mu = h_s/\lambda_s$, where λ_s is the wavelength calculated from the linear dispersion relation with h_s and T_0) is greater than $\delta/\mu = a_0\lambda_s/h_s^2 = 4$ (Mason and Keulegan, 1944). Further, the Ursell number ($\mathrm{Ur} = a_0\lambda_s^2/h_s^3$) provides a more appropriate parameter for harmonic generation (Mei and Unluata, 1972). Given a condition of Ur = 32.9, 25% of the incident wave energy can be converted to the higher harmonics with the second-harmonic amplitude up to 60% that of primary wave

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(Grue, 1992). In addition, the recursion length increases with the period and decreases with the amplitude of the incident conditions (Buhr-Hansen and Svendsen, 1974; Chaplain et al., 1992).

The geometry (i.e., width and/or height) of the obstacle also can be a key factor affecting harmonic generation (Dattatri et al., 1978). For the same incident condition, the obstacle with an increasing width gives insignificant contribution to the harmonic magnitude (Christou et al., 2008). On the other hand, an apparent growth of harmonic amplitudes can be found as the obstacle height increases (i.e., the submergence depth decreases). Once the shallow-water condition ($h_s/\lambda_s < 0.05$) is achieved, the amplitude of the second harmonic can exceed that of the primary wave (Williams, 1964). Besides, the recursion length of the second harmonic increases with the height of the obstacle (or the shallower depth). If the recursion length is about twice the obstacle width, the second-harmonic amplitude reaches its maximum at the trailing edge of the obstacle and further complicates the wave field in the deep-water region behind (Ohyama and Nadaoka, 1994).

Earlier studies have separately established strong fundamental knowledge for the incident and bathymetric/geometric effects on harmonic generation (e.g., see Mei and Unluata, 1972; Dattatri et al., 1978; Ohyama and Nadaoka, 1994). In particular, the characteristics of harmonic generation under highly nonlinear conditions are desirable. Based upon the thorough literature review, it was found that there are three interesting issues worthy of a deeper investigation: (1) A set of new parameters that combine the incident wave and bathymetric effects rather than employ the measured or estimated wave parameters on the top of the obstacle (Grue, 1992; Brossard et al., 2009) may be derived to predict harmonic generation and help engineering practical planning or design; (2) The recursion length obtained by flume experiments and numerical models may deviate 10–30% from the theoretical value computed using the linear dispersion relation (Chaplain et al., 1992; Ohyama and Nadaoka, 1994; Li and Ting, 2012); (3) The distributions of harmonic amplitudes in most experimental and numerical results (e.g., Chaplain et al., 1992; Ohyama and Nadaoka, 1994) show that the minimum of the primary wave ($a_{1\min}$) and the maximum of the second harmonic ($a_{2\max}$) generally occur at different locations, unlike a coincident position described in the second-order Stokes theory (e.g., Massel, 1983).

The purpose of this study is to examine the harmonic generation and associated wave height modulation of nonlinear wave passing over a submerged step. Harmonic generation promoted by the submerged breakwaters is regarded as a key mechanism to maintain the stability/safety of coastal structures (e.g., Losada et al., 1997), since wave energy transferring/spreading to the high-frequency components diminishes the impulse of incident waves (e.g., Vilchez et al., 2016). In this paper, we further clarify the combined effects from various incident wave and bottom geometry conditions for characterizing harmonic generation. A series of flume experiments equipped with the non-intrusive imaging measurement system (Kuo et al., 2009; Li and Ting, 2012) were conducted, covering several incident amplitudes, periods, and bottom configurations (the design parameters of submerged breakwaters, e.g., step widths and step heights). Both 2D Fast Fourier and 1D Morlet wavelet transforms were employed to extract the information of harmonic amplitudes (Ma et al., 2011; Li and Ting, 2012). A new set of parameters (i.e., the Ursell number, relative width, and relative height or depth) was proposed to depict harmonic generation. Also, the third-order Stokes dispersion relation (Chawla and Kirby, 2002) and the concept of energy flux were adopted to better understand the wave height modulation (recursion) phenomenon.

2. Experimental setup and image processing

A series of experiments were conducted in a 20 m(L) × 0.8 m(H) wave flume with a working water depth $h = 27.5$ cm (as shown in Fig. 1). A piston-type wave maker with feedback displacement sensors is equipped at the front end of the tank, whereas a 1:10 slope is constructed at the other end to reduce the reflected waves returning towards the measurement area. The rectangular step is placed 700 cm away from the wave maker with the coordinate origin located at the still water level above, where X is the axis directing toward the end of the tank and Z is the vertical axis. The experiments of Li (2013) preliminarily confirmed that the step width (i.e., $B = 164$ and 328 cm) does not significantly affect the harmonic generation. Along with the experimental results of Li (2013), this study further considered the effects of step height to achieve a complete investigation on the harmonic generation criterion. Various obstacle sizes were used with the width ranging from 250 to 492 cm and the height ranging from 12.4 to 21.0 cm (i.e., the submergence depth $h_s = 6.5$–15.1 cm). Note that the step height (d) and the water depth at step crest (h_s) are complementary in the present study whereas they can be independent in practice/principle. Dimensional analysis will be useful, giving the dimensionless parameter of relative water depth (h_s/h) in a range of 0.24–0.55. The sinusoidal incident waves with different amplitudes (0.15–1.89 cm) and frequencies (i.e., 0.8 and 1.0 Hz) were created using a personal computer. A total of 35 experiment conditions including 15 groups from Li (2013) are tabulated in Table 1, where B is the width of the submerged obstacle, h_s is the water depth above the step (while d is the step height), f and a_0 are the frequency and amplitude of incident wave, respectively, λ is the wavelength based on the linear dispersion relation.

A non-intrusive measurement system is used to record the free-surface elevations. The system comprises a 3 W green-color diode laser with 532 nm wavelength, a set of optical lenses (i.e., a spherical

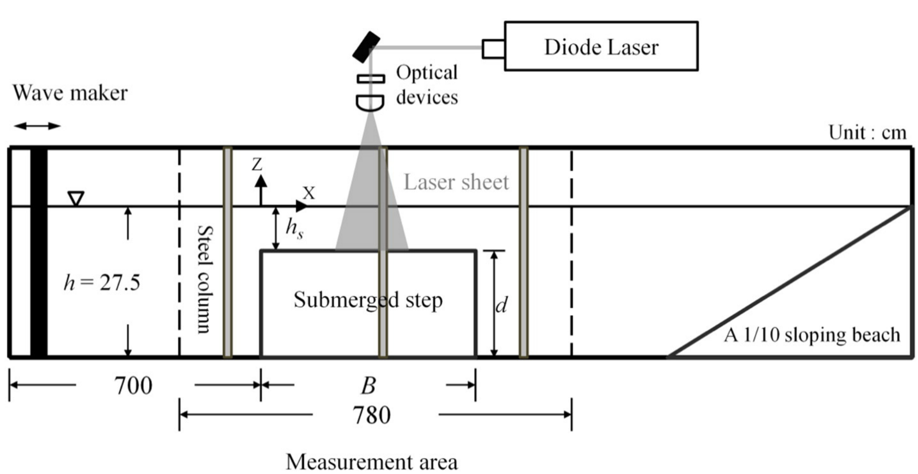

Fig. 1. Wave flume and experimental setup.