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Analytical solutions for estimating tsunami propagation speeds

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ABSTRACT

Recent studies suggest that the tsunami speed can be slowed down by around 1% due to Earth elasticity, water compressibility and density stratification. Analytical solutions of wave dispersion relationship, accounting for such effects, were found in previous studies. In this paper, we investigate the additional effects of water viscosity, ocean stratification due to temperature/salinity and numerical dispersion. Theoretical solutions are derived and checked with known solutions. All the formulas are then simplified for long tsunamis waves so that the propagation speed can be calculated explicitly. The simplified solutions are evaluated using realistic geophysical parameters. For a typical tsunami wavelength of ~200km, the viscous effect is found to be negligible; ocean stratification due to temperature/salinity causes significant speed reduction because of the high density change rate, which has been ignored before. We also evaluate the numerical dispersion of tsunami simulations, which is shown to be potentially comparable to physical dispersion.

1. Introduction

From recent large tsunami events it has been observed that there exists a systematic delay of arrival time compared to the prediction of shallow water equations (e.g., Rabinovich et al., 2011; Wei et al., 2008; Hébert et al., 2009; Saito et al., 2010; Kato et al., 2011; Fujii and Satake, 2013; Kimura et al., 2013; Watada et al., 2014). Watada et al. (2014) summarized the observations and showed that the discrepancy between observation and prediction, which has the order of around 1%, can be explained by the effects of Earth elasticity, water compressibility and geopotential variations. Previous studies have led to the theoretical solutions accounting for one or more of those effects. Mallard et al. (1977) studied the problem of water waves propagating over an elastic bed, using a model that consists of a single layer of incompressible potential fluid over half-space homogeneous elastic earth. The dispersion relation of the small amplitude water waves in such a model was given. Dawson (1978) reviewed the problem but took into account the solid inertia and suggested its importance for cases that include thick soft sediment. Dalrymple and Liu (1978) studied the problem of viscous water waves propagating over a mud bottom using a perturbation method, and derived the dispersion relation and the decay rate of wave amplitude. Ward (1980) presented the theory of tsunami generation and propagation on a spherically symmetric, self-gravitating, elastic Earth in terms of normal modes. Okal (1982) studied the asymptotic behavior of the gravity modes of an incompressible spherical oceanic layer surrounding a rigid Earth, as its radius goes to infinity. Okal showed that the flat-layered ocean tsunami solution and its dispersion is an asymptotic limit of the normal modes of a spherical oceanic shell and that only one branch of tsunami modes exists. Comer (1984) considered water waves in an incompressible fluid within a uniform gravitational field overlying an elastic earth in which the gravitational forces are ignored. Comer concluded that elastic forces are far more important than inertia and they also derived the dispersion relation for the water waves when the Poisson's ration of the Earth is 0.25. Panza et al. (2000) derived solutions for a model with multi-layered compressible inviscid fluid on top of a multi-layered solid half-space Earth, with the compressibility of the fluid treated as elastic solid. Tsai et al. (2013) derived theoretical tsunami propagation speeds accounting for Earth elasticity and water compressibility and stratification based on a method of conservation of potential and kinetic energy. Watada (2013) derived the theoretical dispersion relationship accounting for water compressibility and ocean stratification due to compression of gravity. Allgeyer and Cummins (2014) conducted numerical simulations to

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investigate the effects of elastic Earth. Watada et al. (2014) investigated the data from the 2010 Chilean and 2011 Tohoku tsunamis and showed that the systematic arrival time delay could be explained by the effects of Earth elasticity, water compressibility and geopotential variations.

In this study, we use a method which is slightly different from Watada (2013) to investigate additional secondary effects on tsunami propagation speeds, i.e., the water viscosity and ocean stratification due to temperature/salinity. For completeness the known second order effects, such as the Earth elasticity and water compressibility will be included in the solutions. In addition, we simplify all the theoretical dispersion relationships under the assumption of long waves so that the tsunami propagation speeds can be calculated explicitly. The simplifications are verified for typical tsunami wavelength of 50–1000 km (or wave period 260–5000 s) using realistic geophysical parameters. The relative importance of the second order processes in affecting the tsunami propagation speeds are assessed. Finally, the numerical dispersions of typical tsunami simulating algorithms are evaluated and compared with the above physical effects.

2. Governing equations and boundary conditions

We consider 2D problems in x and z, with x in the horizontal direction to the right and z pointing upwards (Fig. 1).

2.1. Fluid

The linearized governing equations for compressible fluids are derived from the complete Navier-Stokes equations by assuming slight compressibility. They are written as:

$$\begin{cases} \frac{\partial p_1}{\partial t} + B\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) - \rho_0 g w = 0 \\ \rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p_1}{\partial x} + 2\mu\left[\frac{\partial^2 u}{\partial x^2} - \frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right] + \frac{\partial \sigma_{xz}}{\partial z} \\ \rho_0 \frac{\partial^2 w}{\partial t^2} = \left(\frac{\rho_0^2 g^2}{B} + g\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right) w - \frac{\rho_0 g}{B}\frac{\partial p_1}{\partial t} - \frac{\partial^2 p_1}{\partial t \partial z} + \frac{\partial^2 \sigma_{xz}}{\partial t \partial x} + \frac{\partial^2 \sigma_{zz}}{\partial t \partial z} \\ \sigma_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \sigma_{zz} = 2\mu\left[\frac{\partial w}{\partial z} - \frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right] \end{cases} \quad (1)$$

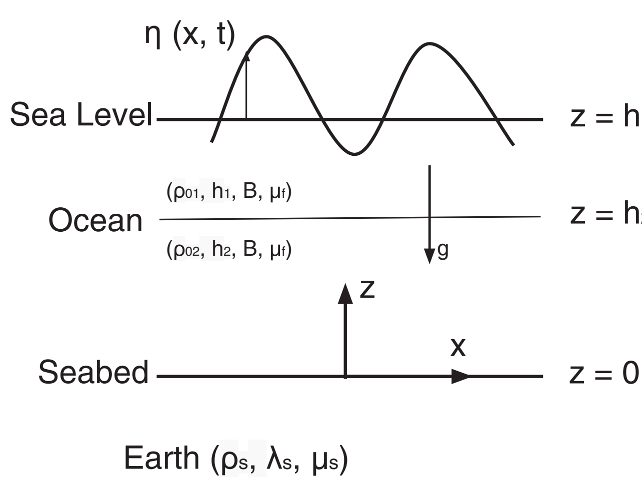

Fig. 1. The 2D model: a tsunami wave $\eta(x,t)$ is propagating over an elastic bed. The z axis origins at the seabed and points upwards; the x axis is in the horizontal direction. The ocean is assumed to be stratified with two layers, with the background density and water depth denoted as (ρ_{01}, h_1) and (ρ_{02}, h_2), respectively. The total water depth $h = h_1 + h_2$. The fluid is assumed to be viscous and slightly compressible, with constant bulk modulus B and viscosity μ_f; Earth is elastic with Lamè constants λ_s and μ_s. The constant gravitational acceleration is g.

where $u(x,z,t)$ and $w(x,z,t)$ are horizontal and vertical velocities, σ_{xz} and σ_{zz} are components of the stress tensor, μ_f is the fluid viscosity and B is the bulk modulus of the fluid. The subscript f of μ_f is omitted for simplicity. The subscript is used to distinguish between the viscosity in the fluid μ_f and the shear modulus in the solid μ_s only when necessary. The total pressure p is defined as the sum of static pressure p_0 and dynamic pressure p_1: $p(x,z,t) = p_0(z) + p_1(x,z,t)$; and the total density ρ is defined as the sum of the background density ρ_0 and density perturbation ρ_1 due to pressure change: $\rho(x,z,t) = \rho_0(z) + \rho_1(x,z,t)$. It is required that $\mathrm{d}p_0(z)/\mathrm{d}z = -\rho_0(z)g$.

Note that it has been assumed that the bulk modulus is constant and that the fluid is only slightly compressible, i.e., $\rho_1(x,z,t) \ll \rho_0(z)$ and $B = \rho\mathrm{d}p/\mathrm{d}\rho$, so we have $\mathrm{d}\rho/\mathrm{d}t = (\rho/B)\mathrm{d}p/\mathrm{d}t$, which leads to, by keeping only the leading order terms,

$$\frac{\partial \rho_1}{\partial t} = \frac{\rho_0}{B}\frac{\partial p_1}{\partial t} - \frac{\rho_0^2 g}{B} w - \frac{\mathrm{d}\rho_0}{\mathrm{d}z} w. \quad (2)$$

The total stress tensor is expressed as

$$\begin{aligned} \mathbf{s} &= -(p_0 + p_1)\mathbf{I} + \begin{bmatrix} \sigma_{xx} & \sigma_{xz} \\ \sigma_{xz} & \sigma_{zz} \end{bmatrix} \\ &= -(p_0 + p_1)\mathbf{I} + 2\mu \begin{bmatrix} \frac{\partial u}{\partial x} - \frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) & \frac{\partial w}{\partial z} - \frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) \end{bmatrix}, \end{aligned} \quad (3)$$

where $\mathbf{I}$ is the identity tensor. The equations are similar to those proposed by Watada (2009), but the shear stress components are included in order to evaluate the viscous effects.

Adopting a plane wave solution in x direction, we have

$$\begin{cases} \eta(x,t) = A\, \mathrm{e}^{i(kx-\omega t)} \\ u(x,z,t) = \hat{u}(z)\, \mathrm{e}^{i(kx-\omega t)} \\ w(x,z,t) = \hat{w}(z)\, \mathrm{e}^{i(kx-\omega t)} \\ p_1(x,z,t) = \hat{p}_1(z)\, \mathrm{e}^{i(kx-\omega t)} \\ \sigma_{xz}(x,z,t) = \hat{\sigma}_{xz}(z)\, \mathrm{e}^{i(kx-\omega t)} \\ \sigma_{zz}(x,z,t) = \hat{\sigma}_{zz}(z)\, \mathrm{e}^{i(kx-\omega t)}, \end{cases} \quad (4)$$

where A, k and ω are the constant wave amplitude, wave number and angular frequency respectively. Substituting the assumed solution (4) into the governing Eq. (1), we obtain a set of first order differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{bmatrix} \hat{u}(z) \\ \hat{w}(z) \\ \hat{p}_1(z) \\ \hat{\sigma}_{xz}(z) \end{bmatrix} = \begin{bmatrix} 0 & -ik & 0 & \frac{1}{\mu} \\ -ik & \frac{\rho_0 g}{B} & i\,\frac{\omega}{B} & 0 \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 2k^2\mu - i\rho_0\omega & i\,\frac{2k\mu\rho_0 g}{3B} & ik - \frac{2k\mu\omega}{3B} & 0 \end{bmatrix} \begin{bmatrix} \hat{u} \\ \hat{w} \\ \hat{p}_1 \\ \hat{\sigma}_{xz} \end{bmatrix} \quad (5)$$

and

$$\hat{\sigma}_{zz}(z) = -i\,2k\mu\,\hat{u}(z) + \frac{4\mu\rho_0 g}{3B}\hat{w}(z) + i\frac{4\mu\omega}{3B}\,\hat{p}_1(z),$$