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Wave-averaged properties in a submerged canopy: Energy density, energy flux, radiation stresses and Stokes drift

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ABSTRACT

This work analyses basic wave properties originating from the interaction between waves and submerged rigid vegetation. First of all, an analytical framework is presented that describes the propagation and dissipation of waves over a rigid and submerged canopy, where the flow resistance is linearised. A nonlinear closure term is introduced to ensure that the work done by the linearised flow resistance equals that of the nonlinear flow resistance. The anisotropic flow resistance is found to have an impact on both the distribution of velocities and pressure inside the canopy and it partly explains the small decrease in flow velocities inside the canopy, which was previously observed experimentally.

The following second order wave properties are derived: the wave energy density, the wave energy flux, the vegetated group velocity of the wave energy density, the radiation stress components parallel and perpendicular to the direction of wave propagation, the Eulerian and Lagrangian Stokes velocities and fluxes. The additional Stokes drift due to the discontinuity in the velocity field at the top of the vegetation is derived; the inclusion of this mass flux in the Lagrangian formulation of the Stokes drift is important for the ratio between the Lagrangian and Eulerian Stokes drifts.

The relation between the wave energy density and the wave energy flux, i.e. the vegetated group velocity of the wave energy density, is of practical importance for large scale wave modelling. The modification to the vegetated group velocity relative to that derived from linear wave theory on non-dissipative waves is described. It is seen that the corrections to linear wave theory are of H^{γ}, where γ is in the interval 1.5–2.0.

1. Introduction

This work was initially inspired by some previous research efforts. First of all, Döbken (2015) showed with the help of a simple 1DV numeric model (Uittenbogaard and Klopman, 2001; Dijkstra and Uittenbogaard, 2010) that the vertical distribution of the Lagrangian Stokes drift velocity in the presence of vegetation differs from that of non-dissipative linear wave theory. The most prominent discrepancy was the local maximum of the Stokes drift velocity adjacent to the top of the canopy, which was caused by the local and large vertical gradient in the horizontal velocity.

Furthermore, (Luhar et al., 2010; Van Rooijen et al., 2016) utilised expressions for the Stokes drift in discussions of the mean flow properties inside a canopy and for the numerical modelling of the wave-induced mean setup inside the Generalised Lagrangian Mean framework (Andrews and McIntyre, 1978). These works relied on a magnitude of the Stokes drift obtained from non-dissipative, linear wave theory.

These combined research efforts led the author to ask the questions: (i) Is it possible to derive an analytical expression for the Lagrangian Stokes drift that also has a local maximum around the top of the canopy? (ii) Based on these results, can it be stated that the Stokes drift is of the same magnitude with and without the presence of vegetation? These questions will be answered in the following.

With an analytical solution at hand, it was straightforward to extend the analysis to additional second order wave properties. The derivation and application of an expression for the radiation stress component in the direction of wave propagation was already reported in Mendez et al. (1998), which is the reason that the focus in the present work is given to the wave energy density, the wave energy flux and the related vegetated group velocity.

Large scale numerical modelling of the interaction between waves and vegetation is conducted over several decades (Dalrymple et al., 1984; Mendez and Losada, 2004; Suzuki et al., 2011; Cao et al., 2015; Van Rooijen et al., 2016). These works cover methods such as the mild slope equations and the wave action equations with

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the common feature that the vegetated group velocity for the wave energy density is set equal to the group velocity:

$$c_g = \frac{1}{2}\frac{\sigma}{k_{\alpha=1}}\left(1 + \frac{2k_{\alpha=1}h}{\sinh 2k_{\alpha=1}h}\right) \tag{1}$$

where $k_{\alpha=1}$ is the wave number, σ is the cyclic frequency and h is the total water depth. The relationship between $k_{\alpha=1}$ and σ is given through the linear dispersion relation for a water depth of h (see Eq. (20) below). α is defined below.

Gu and Wang (1991) saw that the general wave number over a permeable sea bed with isotropic resistance differs from $k_{\alpha=1}$. Consequently, it is natural to ask the following: Since the wave number is a function of the resistance properties of vegetation or a permeable sea bed, will the transport of the wave energy density still be conducted with the group velocity c_g? This leads to related questions concerning the influence of the vegetation on the wave energy density and the wave energy flux. These questions will be addressed in the following.

It is noted that analytical solutions to wave propagation over permeable layers and vegetation were presented previously (Liu and Dalrymple, 1984; Gu and Wang, 1991; Méndez et al., 1999, to name some), but except for the brief outline in Mendez et al. (1998), the second order (wave-averaged) wave properties do not seem to have been given any attention. The same mathematical approach was applied in the works mentioned above for both permeable bed and vegetated fields. It means that the present work is not exclusively limited to vegetated fields, but is also relevant for submerged, permeable breakwaters or permeable natural reefs.

The outline of the present work is as follows. In Section 2 the mathematical framework is presented and the handling of the dissipation due to an anisotropic resistance term is described. A simple validation against experimental data is presented in Section 2.5. In Section 4 the vertical variation of velocities and pressure is discussed along with an example of the phase lags between the horizontal and vertical velocity components. In Section 5 expressions for the wave energy density, wave energy flux and vegetated group velocity are derived and quantified as functions of wave and canopy properties. The radiation stress components along and perpendicular to the direction of wave propagation are presented in Section 6. The horizontal Stokes drift is evaluated in both Eulerian and Lagrangian frameworks in Section 7 and the vertical Stokes velocity at the free surface is utilised to link the Eulerian and Lagrangian expressions. The paper is finalised with a discussion and a conclusion.

2. Mathematical description

The local behaviour of a non-breaking wave field in a canopy consisting of rigid, submerged vegetation is described in this section. The term local means that effects of the finite length of the canopy will not be covered here. A finite length was included in the work by Méndez et al. (1999), and their solution resulted in an expression for the degree of reflection due to the presence of the canopy (though not explicitly analysed). This came with a considerable increase in the complexity of the mathematical description, since evanescence modes were required to match the solution at the ends of the canopy. Consequently, the effects of reflected waves and evanescence modes are omitted in this work. The omission of the evanescence modes are acceptable, since they decay exponentially away from the ends of the canopy (Méndez et al., 1999). The reflected wave is omitted in this work, because the reflection coefficient is assumed small and only second order properties in the wave height are analysed in this work.

Furthermore, boundary layer effects on top of the vegetation field and at the bottom are assumed to be negligible in terms of the wave dissipation. Consequently, these boundary layers will be excluded in this mathematical treatment. Findings by Liu and Dalrymple (1984) for percolation in permeable beds showed that the dissipation due to boundary layers is small in comparison to the dissipation by the permeable medium. This finding is also assumed to hold for dissipation in a submerged canopy.

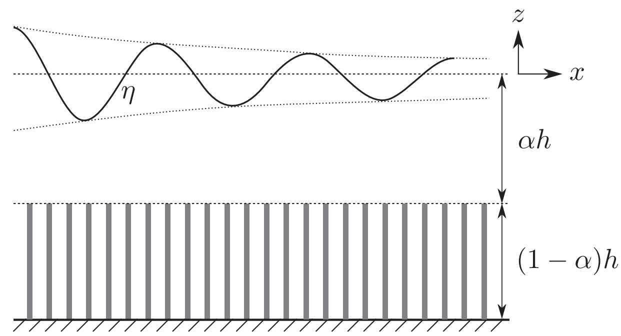

Fig. 1. Sketch of the physical problem of wave propagation and wave attenuation over rigid, submerged vegetation.

Rigid vegetation is not the only type of vegetation, but inclusion of flexibility would greatly increase the complexity of the mathematical derivations. Furthermore, the concept of *effective length* of the vegetation as discussed by Luhar and Nepf (2016) points in the direction of an engineering treatment of the vegetation as stiff elements. The validity of the assumptions will be discussed in Section 8.1.

In the following a vector notation will be used for a matter of compactness of the equations, while a scalar form is used in the subsequent sections. Therefore, the two-dimensional velocity vector is defined as $\mathbf{u} = [u; w]$ and the two-dimensional Cartesian coordinate as $\mathbf{x} = [x; z]$. Lowercase, bold symbols refer to vector properties and uppercase, bold symbols refer to tensors properties of rank 2.

2.1. Definition of the mathematical framework

A sketch of the physical system is presented in Fig. 1. The mathematical derivation will loosely follow the ideas in Gu and Wang (1991), Méndez et al. (1999). Only submerged vegetation will be analysed in this work, consequently $0 < \alpha$.

The wave motion is described by the velocity potential Φ above the canopy, where the velocity field is assumed irrotational and incompressible, i.e. the solution to Φ is given by the Laplace equation:

$$\nabla^2\Phi = 0 \quad \text{for} - \alpha h \le z \le 0 \tag{2}$$

The velocity field above the canopy is given as $\mathbf{u} = \nabla\Phi$.

The flow inside the canopy is described by the linearised momentum equation based on filter velocities (see e.g. Jensen et al., 2014):

$$\begin{aligned}\frac{\partial}{\partial t}\frac{\mathbf{u}}{n} &= -\frac{1}{\rho}\nabla p - \mathbf{C}_m\frac{\partial}{\partial t}\frac{\mathbf{u}}{n} - \mathbf{B}\mathbf{u}\|\mathbf{u}\|_2 \\ &\simeq -\frac{1}{\rho}\nabla p - \mathbf{C}_m\frac{\partial}{\partial t}\frac{\mathbf{u}}{n} - \mathbf{F}\mathbf{u}\end{aligned} \tag{3}$$

The second approximation is a linearisation of the nonlinear resistance term as suggested by Sollitt and Cross (1972). The evaluation of the real-valued friction tensor $\mathbf{F}$ is described in Section 2.3. In Eq. (3), $\mathbf{u}$ is the Eulerian filter velocity vector, n is the porosity, t is time, ρ is the uniform density of water, p is the pressure in excess of the hydrostatic, $\mathbf{C}_m$ is the added mass tensor and $\mathbf{B}$ is the resistance tensor for the quadratic flow resistance. Assuming that $\mathbf{u}$ is periodic in time