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An integral treatment of friction during a swash uprush

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ABSTRACT

The effects of bed friction are modelled for the flow near the moving shoreline during the uprush phase of a swash event by extending the Pohlhausen method used by Whitham (1955) to a sloping bed. The tip of the swash near the moving shoreline is treated in an integral sense as a region of uniform velocity, being acted on by the forces of friction, gravity, and the pressure force induced by the frictionless flow behind the swash tip. The bed shear stress is parameterized by using the quadratic dependence on velocity. The theory is compared to data of the shoreline velocity and position in the swash of breaking solitary waves, and the friction coefficient is determined from direct measurements of bed shear stress. The theoretical predictions are in good agreement with the laboratory results in terms of time history of the shoreline velocity and position, as well as the run-up.

1. Introduction

The swash uprush is the landward movement of water after an incident wave reaches the shoreline. During this phase, the shoreline climbs the beach to reach its run-up, which is defined as the vertical distance between the highest position achieved relative to the initial position of the shoreline. Interest in the swash uprush flow stems from the fact that it is responsible for large volumes of sediment transport (*e.g.* review by Brocchini and Baldock, 2008) and that it is the cause of flooding due to beach overtopping (Peregrine and Williams, 2001; Baldock et al., 2012). A commonly made assumption to understand the swash flow considers the travel of a wave onto an impermeable sloping beach of initially quiescent water. As the wave climbs the beach into shallower water, the wave transforms, breaks, and reaches the shoreline where it collapses to drive the swash. The swash flow is thus driven by complex non-linear processes such as growth of wave amplitude, wave breaking, and bore collapse that are still poorly understood.

For numerical solvers, the major challenges to simulating this situation are the non-linear wave transformation and bore collapse processes, as well as the three-dimensional wave breaking and wave-breaking-generated turbulence (*e.g.* Lin et al., 1999; Lynett et al., 2002; Zhang and L-F Liu, 2008). The moving shoreline is also a challenge to model since it is a time-varying boundary condition to the flow (Packwood and Peregrine, 1981; Borthwick et al., 2006; Antuono et al., 2012; Pedersen et al., 2013). On the other hand, laboratory experiments face difficulties in making measurements of the same processes due to shallow flow depths, entrained air, and unsteady dynamics (Cowen et al., 2003; Jensen et al., 2003; Sou and Yeh, 2011).

One of the major unsolved challenges is the dynamics of the leading tip of the swash, *i.e.*, the flow in the vicinity of the moving shoreline after bore collapse. It is known that flow in this region is significantly affected by the bed friction because the front propagates by continuous breaking of the water surface. Experimental data presented in Pujara et al. (2015) showed that 'the swash solution' to the inviscid shallow water equations on a sloping beach (Shen and Meyer, 1963; Peregrine and Williams, 2001) correctly predicts the flow evolution behind the moving shoreline in the swash of breaking waves, but the motion of the shoreline is not correctly predicted. Direct measurements of bed shear stress at the moving shoreline demonstrated that friction significantly afffects the flow in the swash tip leading to shoreline deceleration, and eventually to a reduced run-up. These experimental observations support the assumption made in previous studies of the closely-related dam-break flows (Whitham, 1955; Hogg and Pritchard, 2004; Ancey et al., 2008; Chanson, 2009): friction is a dominant force near the moving front where the water depth goes to zero, whereas inertia and pressure gradients dominate the dynamics away from the front.

Treatments of friction at the leading front of dam-break flows on a horizontal bed (Whitham, 1958; Hogg and Pritchard, 2004) have not yet been extended to the swash uprush despite the similarity between them (Peregrine and Williams, 2001; Pujara et al., 2015). Thus, the first aim of this paper is to extend the integral treatment of the friction-affected front of a dam-break flow on a horizontal bed

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(Whitham, 1955) to the friction-affected tip during a swash uprush on a sloping beach. While the matched asymptotes approach of Hogg and Pritchard (2004) shows more detailed solutions that include the variation of water depth and velocity within the friction-affected front, their leading order solution closely matches that of Whitham (1955). For the swash uprush, since detailed flow measurements in the friction-affected front are not available, we limit our attention to extending only the leading order solution to the swash uprush on a sloping bed.

The second aim this paper is to present and discuss the application of this integral treatment of friction in the swash tip during uprush to predict shoreline motion due to breaking solitary waves. We obtain predictions of shoreline motion that include the effects of bed friction and compare them to experimental data of shoreline position and shoreline velocity. Data of shoreline motion are still rare and comparisons between data and models usually only consider the shoreline position, and not the shoreline velocity. Comparison of the theory to the latter reveals the limitations of shallow water theory at very short times after bore collapse (Whitham, 1958; Yeh et al., 1989).

Previous efforts to consider the effects of friction on shoreline motion (*e.g.* Kirkgöz, 1981; Hughes, 1995; Puleo and Holland, 2001; Svendsen, 2006) have done so by adding a frictional term of the form fu^2/h (with the water depth in the denominator replaced by a constant) to the equation of motion for a fluid parcel that otherwise follows a ballistic motion. However, experimental results have shown that fluid parcels do not follow a ballistic motion, but instead converge at the swash tip (Baldock et al., 2014). Further, Antuono et al. (2012) used asymptotic expansions of the non-linear shallow water equations including a term of the form fu^2/h in the vicinity of the shoreline to show that the shoreline never recedes from its highest position. This non-physical result shows that previous solutions may rely on assumptions that have been cast into doubt. In this paper, we focus only on the uprush and provide an alternative treatment of friction on shoreline motion.

This paper is organized as follows. We show the formulation to derive the equations of motion for the swash tip in Section 2, and show the solutions and comparisons to experimental data in Section 3. Section 4 gives the conclusions.

2. Formulation

The non-linear shallow water equations (NSWE) for flow in the swash zone are written in a co-ordinate system parallel and perpendicular to the beach (see Fig. 1) as

$$\frac{\partial h}{\partial t}+\frac{\partial (hu)}{\partial x}=0, \tag{2.1a}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\cos\theta\frac{\partial h}{\partial x}+g\sin\theta=0, \tag{2.1b}$$

where g is the gravitational acceleration, $u(x,t)$ is the depth-averaged velocity parallel to the beach, $h(x,t)$ is the water depth measured perpendicular to the beach surface, and θ is the beach slope. The NSWE are one-dimensional equations representing the conservation of mass and momentum, respectively, and are known to be a good approximation to the flow motion in the swash zone (Peregrine, 1972) except for the effects of bed friction, which is ignored in these equations. A swash event is driven by the process of bore collapse when a bore or a breaker reaches the shoreline (reviewed in Meyer and Taylor, 1972). The resulting shoreline motion is given by Shen and Meyer (1963) as

$$x_s=U_s t-\frac{1}{2}g(\sin\theta)t^2, \tag{2.2a}$$

$$u_s=U_s-g(\sin\theta)t, \tag{2.2b}$$

where $x_s(t)$ is the shoreline position, $u_s(t)$ is the shoreline velocity, $U_s=u_s(0)$ is the initial shoreline velocity, and $t=0$ denotes the start of the swash. Peregrine and Williams (2001) provided a solution to the NSWE for the entire swash, *i.e.*, $(x,t)>(0,0)$, by extending the asymptotic results of Shen and Meyer (1963). This 'swash solution' gives the flow evolution as

$$h(x,t)=\frac{1}{g}\left[\frac{1}{3}U_s-\frac{1}{6}g(\sin\theta)t-\frac{1}{3}\frac{x}{t}\right]^2, \tag{2.3a}$$

$$u(x,t)=\frac{1}{3}U_s-\frac{2}{3}g(\sin\theta)t+\frac{2}{3}\frac{x}{t}, \tag{2.3b}$$

which is the same as the solution to the NSWE for a dam-break flow on a sloping bed in which the initial water depth behind the dam is $U_s^2/4g$ (Peregrine and Williams, 2001). In this 'swash solution', the shoreline climbs the beach for $g(\sin\theta)t/U_s\leq 1$ and its furthest position along the beach is given by $g(\sin\theta)x/U_s^2=0.5$. Antuono and Hogg (2009) and Antuono (2010) give further details of the inviscid swash flow from similar initial conditions. In the inviscid swash solution described above, the shoreline is approached with the free-surface tangential to the bed, but, in a real flow, frictional effects lead to a 'blunt nose' and flow deceleration so that the actual shoreline position, $a(t)$, is behind the frictionless shoreline solution, $x_s(t)$, as indicated in Fig. 1. If friction with the bed only alters the dynamics in the region immediately behind the moving shoreline, which we term the 'swash tip' region, then denoting $\xi(t)$ as the interface between the friction-affected swash tip and the frictionless flow, $(a-\xi)$ becomes the extent of the swash tip region in the x-direction. The flow behind $\xi(t)$ is frictionless and follows the swash solution, Eq. (2.3).

Treating the swash tip in an integral sense (*cf.* Brocchini and Peregrine, 1996; Archetti and Brocchini, 2002; Brocchini, 2006), the conservation of mass is written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\xi(t)}^{a(t)}\rho h(x,t)\mathrm{d}x=\rho h(\xi(t),t)\left[u(\xi(t),t)-\frac{\mathrm{d}\xi(t)}{\mathrm{d}t}\right], \tag{2.4}$$

which can be integrated in time using Eq. (2.3) to express $h(\xi(t),t)$ and $\xi(t)$ in terms of $u(\xi(t),t)$:

$$\xi(t)=\frac{3}{2}u(\xi(t),t)t-\frac{1}{2}U_s t+g(\sin\theta)t^2, \tag{2.5}$$

$$h(\xi(t),t)=\frac{1}{4g}[U_s-u(\xi(t),t)-g(\sin\theta)t]^2. \tag{2.6}$$

Inserting Eqs. (2.5)–(2.6) in Eq. (2.4), and using the initial condition $\left[\int_{\xi(t)}^{a(t)}\rho h(x,t)\mathrm{d}x\right]=0$ at $t=0$, gives

$$\left[\int_{\xi(t)}^{a(t)}\rho h(x,t)\mathrm{d}x\right]=\frac{\rho t}{8g}[U_s-g(\sin\theta)t-u(\xi(t),t)]^3. \tag{2.7}$$

The integral momentum equation for the swash tip is written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\xi(t)}^{a(t)}\rho h(x,t)u(x,t)\mathrm{d}x=\rho h(\xi(t),t)u(\xi(t),t)\left[u(\xi(t),t)-\frac{\mathrm{d}\xi(t)}{\mathrm{d}t}\right]+F, \tag{2.8}$$

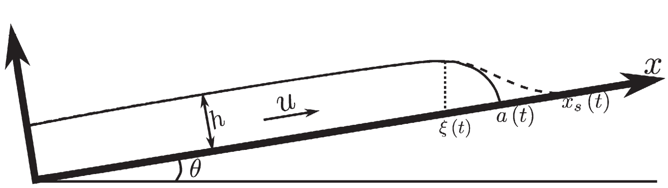

Fig. 1. Definition sketch.

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