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Comparative study of joint distributions of wave height and surf parameter for individual waves including spectral bandwidth effects

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ABSTRACT

This paper compares the Myrhaug and Fouques (2012) joint distribution of wave height and surf parameter with the transformed Longuet-Higgins (1983) joint distribution of wave height and wave period; both joint distributions are for individual random waves within a sea state. Effects of the spectral bandwith are discussed, together with giving examples of estimating probabilities of breaking waves on different slopes. The Myrhaug and Fouques (2012) distribution is a parametric model originating from best fit to relatively broad-banded field data, while the Longuet-Higgins (1983) distribution is theoretically based. It is found that the theoretically based distribution is inadequate to describe the features of the parametric model representing the relatively broad-banded data, suggesting that parametric models should be used to describe such data.

1. Introduction

The surf parameter, also often referred to as the surf similarity parameter or the Iribarren number, is used to characterize surf zone processes. It is given by the ratio between the slope of a beach or a structure and the square root of the wave steepness in deep water as introduced by Iribarren and Nogales (1949) and used later by Battjes (1974) (see definition in Section 2). Shallow water regions where waves break are referred to as the surf zone, and the different breakers on slopes are defined and classified in terms of the surf parameter (see e.g. Battjes (1974)). The surf parameter also enters in many empirical and theoretical models for wave-induced phenomena in the surf zone; wave-breaking is associated with large loss of energy, also resulting in strong currents along the shoreline and thereby affecting the nearshore circulation; along beaches the wave energy flux from offshore is dissipated into turbulence and heat, and finally the wave height decreases towards the shoreline. The high intensity of turbulence caused by wave-breaking is also responsible for the intense sediment transport in the surf zone. Wave runup on beaches and coastal structures such as e.g. breakwaters, seawalls and artificial reefs are characterized by using the surf parameter. The surf parameter is commonly defined in terms of individual wave parameters, but a characteristic surf parameter defined in terms of sea state parameters is also used. Examples of the relevance and importance of the surf parameter are found in e.g. Silvester and Hsu (1997); Kim (2010). Recent examples of using a surf parameter defined in terms of sea state parameters to estimate characteristic values of the wave runup are given by e.g. de la Pena et al. (2014) and Blenkinsopp et al. (2016); also including literature reviews. Moreover, the joint statistics of the surf parameter with wave heights, or the surf parameter with wave heights above a specified threshold may be appropriate in formulating risks of e.g. damage of breakwaters, seawalls and artificial reefs. This is the case for surf parameters defined for both individual waves and for sea states.

Previous works on statistical aspects of the surf parameter for individual waves include Tayfun (2006); Myrhaug and Fouques (2007); Myrhaug and Rue (2009) and Myrhaug and Leira (2011), while Myrhaug and Fouques (2010) considered the surf parameter for sea states. A brief review of these works was given in Myrhaug and Fouques (2012), which provided bivariate distributions of the surf parameter and the wave height as well as the breaker index and the wave height for individual random waves within a sea state. Their results were obtained by transformation of a joint distribution of wave steepness and wave height proposed by Myrhaug and Kjeldsen (1984), which is a parametric distribution obtained as best fit to data from a larger data base collected at deep water locations of the Norwegian continental shelf. These data, and thus the parametric model, are relatively broad-banded representing real sea state waves. Statistical properties of the wave parameters were presented, and the joint distribution was applied to estimate the probability of breaking waves on different slopes for all wave heights, and for wave heights exceeding the significant wave height. An example of calculating the vertical wave runup on slopes corresponding to typical field conditions was also provided. Myrhaug (2015) presented a joint distribution of

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significant wave height and wave runup by adopting de la Pena et al.'s (2014) runup formulation. Estimates of wave runup on shorelines were also exemplified.

The purpose of this paper is to compare the Myrhaug and Fouques (2012) joint distribution of wave height and surf parameter with the transformed Longuet-Higgins (1983) joint distribution of wave height and wave period for individual random waves. Spectral bandwith effects are discussed, which are included in the Longuet-Higgins distribution. Examples of estimating the probability of breaking waves for all wave heights on different slopes are also given.

2. Background

The surf parameter is defined as $\xi = m/\sqrt{s}$ where $m = \tan\theta$ is the slope with an angle θ with the horizontal, $s = H/((g/2\pi)T^2)$ is the wave steepness in deep water, H is the wave height in deep water, T is the wave period, and g is the acceleration due to gravity. The surf parameter is normalized, i.e. $\hat{\xi} = \xi/\xi_{rms}$, by defining $\xi_{rms} = m/\sqrt{s_{rms}}$ where s_{rms} is a normalization factor of s, which will be given in the forthcoming.

This paper considers two joint probability density functions (*pdfs*) of surf parameter and wave height; one is based on transformation of the theoretical Longuet-Higgins (1983) (hereafter referred to as LH83) joint *pdf* of H and T; the other is based on the parametric Myrhaug and Fouques (2012) (hereafter referred to as MF12) joint *pdf* of ξ and H. In the following the background of the LH83 *pdf* is given in Section 2.1, while the MF12 *pdf* is given in Section 2.2.

2.1. LH83 distribution

The LH83 joint *pdf* of H and T is chosen to obtain the joint *pdf* of surf parameter and wave height including the effect of spectral bandwidth. LH83 was derived by considering the statistics of the wave envelope phase. This distribution is based on a narrow-band approximation.

Table 1
Peak values and their locations for the MF12 *pdf* and the transformed LH83 *pdf*.

Distribution	ν	$\hat{h}$	$\hat{\xi}$	Peak value
MF12	–	0.905	1.120	1.160
Transformed LH83	0.1	1.105	0.940	4.362
Transformed LH83	0.3	1.075	0.930	1.516
Transformed LH83	0.504	1.030	0.915	0.974
Transformed LH83	0.6	1.005	0.905	0.855

The LH83 joint *pdf* of wave height and wave period is given as

$$p(h,t) = \frac{2L(\nu)}{\nu\sqrt{\pi}}\left(\frac{h}{t}\right)^2 \exp\left\{-h^2\left[1+\frac{1}{\nu^2}\left(1-\frac{1}{t}\right)^2\right]\right\} \quad (1)$$

where

$$h = \frac{H}{2\sqrt{2m_0}} \quad (2)$$

$$t = \frac{T}{2\pi\frac{m_0}{m_1}} \quad (3)$$

are the dimensionless wave height and wave period, respectively, and

$$L(\nu) = \frac{2}{1+\left(1+\nu^2\right)^{-1/2}} \quad (4)$$

$$\nu^2 = \frac{m_0 m_2}{m_1^2} - 1. \quad (5)$$

Here m_n is the spectral moments defined as

$$m_n = \int_0^\infty \omega^n S(\omega)d\omega \;;\; n = 0,1,2,--- \quad (6)$$

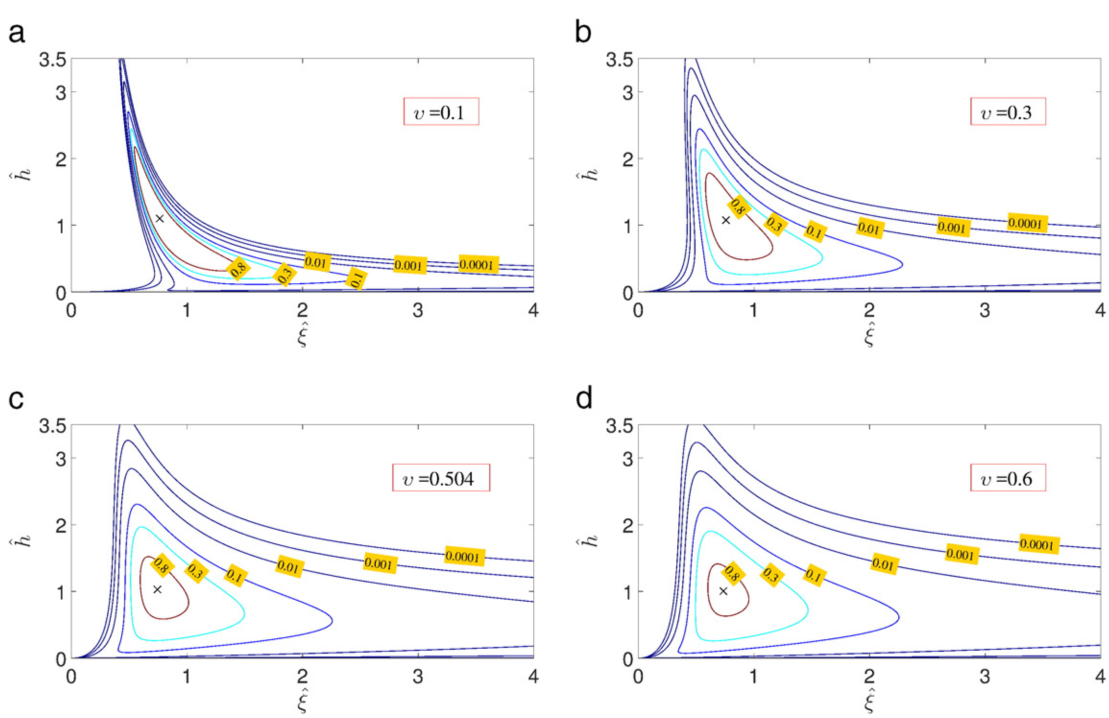

Fig. 1. Isocontours of the transformed LH83 $p(\hat{h},\hat{\xi})$: (a) $\nu = 0.1$; (b) $\nu = 0.3$; (c) $\nu = 0.504$; (d) $\nu = 0.6$.