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Can the Rayleigh distribution be used to determine extreme wave heights in non-breaking swell conditions?

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ABSTRACT

A reliable set of tools for prediction of low-exceedance design waves is of high importance when designing coastal protection structures. The significant wave parameters are typically obtained from buoys or numerical wave propagation models and design values are found by extreme analysis. Statistical wave height distributions are used to transform the significant wave height to lower exceedance wave heights. These extreme single waves will cause the highest loads and wave overtopping volumes on structures and thereby represent the design conditions. An under-prediction of the design maximum wave height causes unsafe designs, while an over-prediction causes too conservative and thus expensive designs. The wave height distribution by Longuet-Higgins (1952) (Rayleigh-distribution) for deep-water non-breaking waves is in the present paper evaluated against data from numerical tests with long period and long-crested swell waves. The numerical model is validated against data from physical model tests. Generally, it is concluded that the Rayleigh-distribution is under-predicting the low-exceedance wave heights in irregular swell waves. This is expected to be caused by wave non-linearity and thus a new modified wave height distribution is suggested, where the shape parameter in the distribution is dependent on the wave non-linearity, represented by the Ursell-number. The new proposed wave height distribution for non-linear and non-breaking waves is highly applicable for practical engineering design of both near-shore and offshore structures under influence of swell-waves.

1. Introduction

When designing marine structures the design basis is in most cases only providing the significant wave height and the mean or peak wave period. However, for design of coastal and offshore structures, especially the wave heights with low-exceedance probabilities are of importance, since these lead to the highest wave loads and largest wave run-up heights and wave overtopping volumes on the structures. Higher accuracy in the prediction of the design maximum wave height is thus important for reliable and cost effective designs.

Deep-water waves can be propagated to shallow water using numerical phase-averaging or phase-resolving models. Two-dimensional or three-dimensional phase-resolving wave propagation models can provide surface elevations in the time-domain and are able to transform waves from deep to shallow water with high accuracy. This includes shallow-water wave transformation effects, such as wave shoaling, wave refraction, wave non-linearity, and wave–wave interactions. The phase-resolving models are, however, more computationally demanding and therefore, for practical application, the more computationally efficient phase-averaging models, such as e.g. SWAN (Holthuijsen et al., 1993), are often used to determine the frequency domain wave parameters in shallower water. Many phase-averaging models are still capable of including the same physical wave transformation processes as the phase-resolving models, but if low-exceedance wave heights are needed, an appropriate wave height distribution must be applied in combination with the numerical model results. A reliable wave height distribution is thus needed.

It is in many cases assumed that the Rayleigh distribution is conservative to apply for extreme wave heights. However, the hypothesis in the present paper is that this might not be the case for highly non-linear waves like swells, because high waves shoal more in height than smaller waves, which in non-linear wave conditions causes waves to be higher than given by the Rayleigh distribution. The consequence is similar to the distribution by Forristall (2000) for individual wave crest heights in non-linear waves where the shape parameter, b, in the Rayleigh-distribution for distribution of wave heights is a function of the wave non-linearity. The effect is though expected to be less for wave heights than for crest heights. Suggestions to shape-parameter modifications for wave heights will be given in this paper based on data from numerical model tests with linear to non-linear waves.

This paper evaluates and modifies the Rayleigh-distribution (Longuet-Higgins, 1952) for prediction of low-exceedance wave heights in cases where waves are non-breaking, but become highly non-linear

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due to shoaling. Various seabed slopes and spectral energy distributions are evaluated. The study is based on numerical model results supported by physical model test validation cases.

2. Background

Waves generated from wind are irregular and short-crested. If the waves propagate out of the wind area, they become more regular and close to long-crested (swell waves). When approaching the shore three processes will transform the waves. The first wave transformation process is wave shoaling/refraction. The second wave transformation process is the change from a linear to non-linear surface elevation where crests are narrow and high. The third process is wave breaking, which will dissipate energy of the low-exceedance wave heights and thereby cause changes in the wave height distribution. State-of-art wave height distributions and non-linear wave theories are described in the following together with the expected influence of wave non-linearity on the wave height distribution.

2.1. Wave height distribution in non-breaking wave conditions

Longuet-Higgins (1952) showed that wave heights are theoretically Rayleigh-distributed in the case of a Gaussian-distributed narrow-banded linear surface. Relations were derived between different characteristic wave heights, such as significant wave height, $H_{1/3}$, and mean wave height, H_m, in the time-domain, given in Eqs. (1) and (2).

If individual wave heights in irregular sea follows the Rayleigh-distribution, then the significant wave height based on the time domain ($H_{1/3}$) and frequency domain (H_{m0}) are identical. Thereby many designers use either of the two without any further notice whether they stem from the time- or frequency domain analysis. However, in-situ measurements from real sea conditions with real spectra have shown that $H_{1/3}/H_{m0}$ in average is approximately 0.95 (Goda, 2010), which results in the wave height distribution in Eq. (3).

$$F(H) = 1 - \exp\left(-2\left(\frac{H}{H_{1/3}}\right)^b\right), \quad b = 2 \text{ (i.e.Rayleigh-distribution)} \tag{1}$$

$$F(H) = 1 - \exp\left(-\frac{\pi}{4}\left(\frac{H}{H_m}\right)^b\right), \quad b = 2 \text{ (i.e.Rayleigh-distribution)} \tag{2}$$

$$F(H) = 1 - \exp\left(-2\left(\frac{H}{H_{m0}\cdot 0.95}\right)^b\right), \quad b = 2 \text{ (i.e.Rayleigh-distribution)} \tag{3}$$

2.2. Wave height distribution in breaking wave conditions

Due to conservation of energy from deep to shallower water the wave height increases while the wave group velocity decreases (wave shoaling), which will increase the wave steepness, H/L, and in the end lead to wave breaking. Miche (1944) showed theoretically that the wave particle velocity could not exceed the phase wave velocity, which led to the maximum possible wave steepness given in Eq. (4), where k is the wavenumber.

$$S_{\max} = (H/L)_{\max} = 0.142\cdot\tanh(kh) \tag{4}$$

The upper limit of the H/h-ratio for regular waves can be found based on Eq. (4) to be $H/h \le 0.89\cdot h$. However, a ratio of $H/h \le 0.6$–0.8 (depending on the bed slope) is more commonly used.

The Rayleigh distributions in Eqs. (1), (2), and (3) does not account for wave breaking, and thereby they over-predict the low-exceedance wave heights in these conditions. Mendez et al. (2004) proposed a probability density function (pdf) for the transformation of depth-limited wave height distributions including shoaling and breaking on a planar beach. Additionally, Battjes & Groenendijk (2000) suggested the widely used composite Weibull-distribution, given in Eq. (5), which is now recommended for use in breaking wave conditions in The Rock Manual (CIRIA et al., 2007), the EurOtop Manual (Pullen et al., 2007), and the DNV-OS-J101 standard on design of offshore wind turbine structures (DNV, 2010). H_1 and H_2 are scale parameters and k_1 and k_2 are shape factors. For conditions with non-breaking waves (below a certain transition wave height H_{tr} given in Eq. (5)), the Battjes & Groenendijk (2000) distribution has a shape factor $k_1 = 2$, i.e. a Rayleigh-distribution. In breaking wave conditions the upper tail of the distribution changes to a Weibull-distribution with higher shape factor $k_2 = 3.6$.

Expressions for H_1 and H_2 are given in Battjes & Groenendijk (2000). H_1 and H_2 are set to vary for varying transition wave height. H_{rms} is the root-mean-square wave height, which for linear wave conditions is given as $H_{rms} = (8\cdot m_0)^{0.5}$ where m_0 is the variance of the surface elevation. However, for non-linear waves the ratio is larger and Battjes and Groenendijk (2000) suggested the relation; $H_{rms} = (2.69 + 3.24\cdot m_0^{0.5}/h)\cdot m_0^{0.5}$. Other inputs to the distribution are the foreshore bed slope β_f, h, and H_{m0}.

$$F(H) = \begin{cases} 1 - \exp\left[-(H/H_1)^{k_1}\right] & \text{for } H \le H_{tr} \\ 1 - \exp\left[-(H/H_2)^{k_2}\right] & \text{for } H > H_{tr} \end{cases} \tag{5}$$
$$H_{tr} = (0.35 + 5.8\tan(\beta_f))\cdot h$$

Goda (2012) evaluated the Battjes & Groenendijk (2000)-distribution against laboratory data from Hamm & Pernnard (1997) and noted that k_2 was varying in the range $2.2 < k_2 \le 3.7$ for water depth to deep-water wave height ratios of $2.73 < h/H_0 \le 0.66$. It was concluded, that $k_2 = 3.6$ by Battjes & Groenendijk (2000) corresponds approximately to the middle of the surf zone.

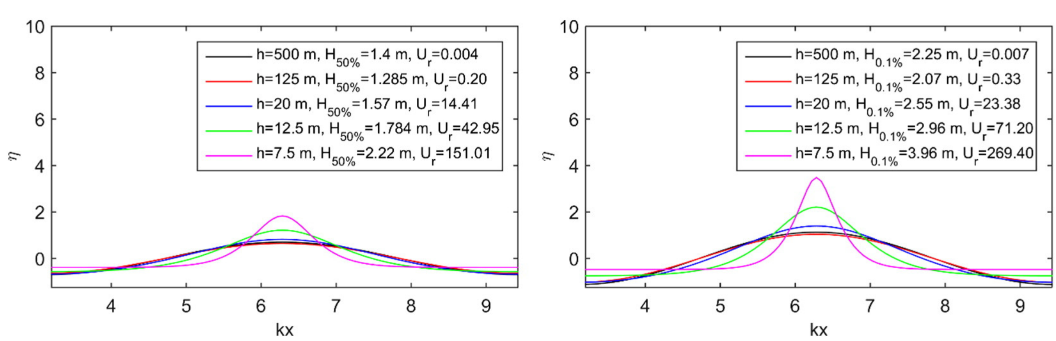

Fig. 1. Surface elevations corresponding to $H_{50\%}$ (*left*) and $H_{0.1\%}$ (*right*) at different water depths calculated based on stream function theory of 30th order.