Coastal Engineering 103 (2015) 42–51

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

On the distribution of significant wave height and associated peak periods

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ARTICLE INFO

Article history:
Received 19 December 2014
Received in revised form 19 May 2015
Accepted 13 June 2015
Available online 20 July 2015

Keywords:
Average conditional exceedance of peak periods
Significant peak period
50 and 100 year peak periods
Erlang distribution
Generalized *Pareto* distribution (*GP*3)
Three-parameter *Weibull* distribution

ABSTRACT

This study uses 21 years (1958–1978) significant wave height and associated peak periods off Azores in the North Atlantic Ocean, extracted from 44 years HIPOCAS database. Empirical average conditional exceedances of peak periods are executed. Plausibility of judging the distribution of the peak periods by modelling average conditional exceedance of peak periods by *Erlang*, generalized *Pareto* and three-parameter *Weibull* models is investigated and we also assessed certain peak period statistics predicted by the models. 50 year gamma peak period quantiles are reasonably accurate when compared with 44 year peak period quantiles (HIPOCAS). *Erlang* and generalized *Pareto* estimate of mean peak period are reasonable; whereas all the three models fairly evaluate the average of the one-third the highest peak periods. *Weibull* model derived parametric relation gauged the average of the one-tenth the highest peak periods. A general statistical formula is suggested for estimation of significant wave period. Average of one-third the highest peak period estimates by the parametric relation derived from generalized *Pareto* distribution using the general formula for significant wave period, provides reliably precise results. Significant wave period to mean wave period observational ratio of 1.2 is appropriately interpreted for both computed and estimated ratios of mean peak period of one-third the highest significant wave heights to mean peak periods.

1. Introduction

The probability density function of wave periods has received less attention in the literature than that of wave heights. This is because significant wave height (H_s) is the most significant parameter influencing the intensity of extreme loads in fixed structures like coastal structures and fixed offshore structures. For floating structures H_s continues being the most important variable, but in this case the average period of the waves influences the dynamic amplification of the response and thus has also an important influence on the wave induced loads.

So, while the mostly static coastal and offshore structures require long term and extreme distributions of significant wave height (e.g., Ferreira and Guedes Soares, 2000; Guedes Soares and Scotto, 2001, 2004) floating structures require bivariate distributions of significant wave heights (H_s) and characteristic (mean or peak) periods (Bitner-Gregersen et al., 1998; Ferreira and Guedes Soares, 2002; Haver, 1985). Significant wave height (H_s) and associated peak period (T_p) are important wave parameters that determine the characteristics of the wave energy spectrum, which is used for assessing the response spectrum to be considered in long term formulations (Guedes Soares and Moan, 1991).

Floating wave energy converters (Falcão, 2010) are normally in the coastal zone in areas of low water depth and their dynamic response depends on the sea states defined by significant wave height (H_s) and associated peak period (T_p) (e.g., Silva et al., 2013). So, while their survivability depends also on the extreme sea states mentioned before, their operational performance depends mostly of the wave period. In fact, a given wave energy converter will have a range of significant wave heights in which it will perform, but its efficiency depends on the response amplification factor, which is governed solely by the mean wave period. Therefore, to study the efficiency of these devices, having a probability distribution of characteristic wave periods is essential.

The marginal distributions of wave periods, even under the assumption that waves are linear and have narrow band frequency spectrum (Rodríguez et al., 2004) are difficult to determine probably because often sea states have a certain percentage of cases with two wave systems (Guedes Soares, 1984), in which case the distribution of periods will appear bimodal and will not fit the distribution appropriate to single sea states. One approach to determining the distribution of wave periods is to derive it as the marginal distribution of the joint distribution of individual wave heights (H) and periods (T). By this strategy, Longuet-Higgins (1975, 1983) obtained the marginal density function of wave period:

$$\boldsymbol{f}(\boldsymbol{t}) = \left(\mathbf{1} + \frac{\boldsymbol{c}^2}{\mathbf{4}}\right)\frac{\mathbf{1}}{\mathbf{2}\boldsymbol{c}\boldsymbol{t}^2}\left[\mathbf{1} + \left(\mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{t}}\right)^2\frac{\mathbf{1}}{\boldsymbol{c}^2}\right]^{-\frac{3}{2}}, \quad \boldsymbol{t} > 0 \tag{1}$$

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http://dx.doi.org/10.1016/j.coastaleng.2015.06.001
0378-3839/

where c — spectral width parameter; $\overline{T}$ — mean wave period; $t = \frac{T}{\overline{T}}$; T — wave period random variable. Arhan et al. (1976) and Kwon and Deguchi (1994) also provided joint distributions of wave heights and periods. Nevertheless, the wave period distributions depend on the shape of the wave spectrum. The theoretical marginal distribution of wave periods given by Cavanié et al. (1976) is of the form:

$$f(t) = \frac{\alpha^3\beta^2 t}{\left[\left(t^2-\alpha^2\right)^2+\alpha^4\beta^2\right]^{\frac{3}{2}}} \tag{2}$$

where $\alpha = \frac{1}{2}(1+\sqrt{1-\varepsilon^2})$; $\beta = \frac{\varepsilon}{\sqrt{1-\varepsilon^2}}$; ε — spectral width parameter.

Subsequent mathematical treatments for estimation of certain wave period statistics of interest from these models are strenuous due to their intricacies. Henceforth, restricted applications of these models persist such as the fitting to observed wave period distribution.

Other joint distribution models of wave heights and periods are provided by Lindgren and Rychlick (1982) and Tayfun (1993). The marginal distribution of wave periods provided by Lindgren and Rychlick is not based on the zero crossing definition of wave periods and constrained by numerical integration and Tayfun's model is only applicable for wave periods associated with waves larger than the mean wave height.

A second and conventional procedure is to fit known probability distributions taking into account the physical features of the wave periods. Wist (2003) modelled successive wave periods with the Nataf transformation using a two-parameter *Weibull* distribution and a generalized gamma distribution, and the latter was in good agreement with the data. Of the various probabilistic models considered for comparison with zero up-crossing wave periods in Gaussian combined sea states, the distribution recommended by Myrhaug and Slaattelid (1999), which is an amalgamation of two, two-parameter *Weibull* functions represents adequately the observed distributions of wave periods (Rodríguez et al., 2004). Anyway, no attempt appeared to have been made to gauge other wave period statistics from these models.

Muraleedharan et al. (2009) determined $m(t)$; the empirical average conditional exceedance of zero crossing wave periods t for a shallow water site (Valiathura, southwest coast of India, depth of recording 5.5 m) in the Arabian Sea for the month of January, 1981. The long-term data of shallow water mean zero crossing wave periods ($\overline{T}_z$) off Valiathura, computed by CESS (1980–1984) for the rough south west rainstorm seasons (May–October) were additionally considered. The discrepancies among computed and estimated (by $m(t)$ derived from *Erlang* distribution) average conditional exceedance of wave periods were insignificant according to the $RRMS_{error}$ criteria. The plausibility of obtaining the gamma (or *Erlang*) as the distribution of wave periods T by modelling the function $m(t)$ has been investigated. Since $m(t)$ resolves the distribution of T uniquely, it is adequate to find the functional form of $m(t)$ coherent with the data. Thus modelling the wave periods can be accomplished through the function $m(t)$, provided sufficiently precise description of the functional form of $m(t)$ is derived from the data. In support of the above assertion, the gamma distribution was successfully fitted to the zero crossing wave period distribution off Valiathura (CESS, India), to the average wave period distribution of a cyclone sea state (June, 2004) recorded by DS1 deep water buoy (15.5°N, 69.25°E) and to data from the shallow water buoy SW3 (15.4°N, 73.8°E) off Goa in the Arabian Sea.

This study also examines the prospects of extending these *regional* aspects by modelling *globally* distributed daily maximum significant wave height (H_s) and associated peak periods (T_p). The distribution of peak period is more crucial than wave heights in assessing the efficiency of wave energy converters. The paper utilises, the month wise grouped 21 years (1958–1978) everyday maximum significant wave height and associated peak period distributions off Azores (Fig. 1, 0.25° × 0.25° grid) in the North Atlantic Ocean (Pilar et al., 2008), which were extracted from 44-years HIPOCAS database. Lucas et al. (2014) detailed about the generation and validity of these data sets. Campos and Guedes Soares (2012) also calibrated and validated the data set with the NOAA\NCEP data sets ensuing that the two data sets are similar in the area of study (20°–60°N, 50°W–04°E), the region that composes offshore Portugal. Outlier analyses (Wilks, 2006) are also performed to identify and discard extreme outliers in the data.

There is sufficient substantiation that environmental parameters are distributed with heavy tails (Ahmad et al., 1988; Ferreira and Guedes Soares, 1999; Houghton, 1978; Landwehr et al., 1979; Rossi et al., 1984). Hosking and Wallis (1997) have suggested that it is astute to utilize an extensive variety of heavy tailed distributions or a distribution with enough free parameters that it can replicate a wide range of frequency distributions. The present study considers the generalized *Pareto* (*GP*3) and 3-parameter *Weibull* distributions along with *Erlang* (or gamma) distribution as candidates of long-term marginal distributions of peak periods (T_p) and applies these models to month-wise clustered joint distributions of daily maximum significant wave heights and associated peak periods off Azores. The defense for the gamma distribution as a possibility for wave period distribution on certain basic assumptions is provided by Unnikrishnan Nair et al. (2003). The method of L-moments (Hosking and Wallis, 1997) is used to estimate the model parameters. L-moments are a recent development in statistics and provide an elegant mathematical theory and facilitate the estimation process. Hosking et al. (1985) and Hosking and Wallis (1987) noted that with small and moderate samples the method of L-moments is often more efficient than maximum likelihood. Furthermore, they demonstrated that the method of L-moments yields computationally advantageous assessments of parameters and quantiles.

Certain peak period statistics such as mean peak period ($\overline{T}_p$), average of one-third the highest peak periods ($\overline{T}_{p(\frac{1}{3})}$), 50 ($Q_{50}$) and 100 ($Q_{100}$) year return peak periods are also estimated and compared with computed values to emphasize the accuracy of the estimations.

Kitano et al. (2002) derived a formula for significant wave period based on complex value integral of energy spectrum and depends on the spectral shape asymmetry. This work also provides a new general statistical formula for estimation of significant wave period (T_s) from time series data. The parametric relation derived (based on general statistical formula for T_s) from the generalized *Pareto* distribution (*GP*3) estimated $\overline{T}_{p(\frac{1}{3})}$ with reasonable accuracy. Empirical relationship between mean period of one-third the highest wave heights $\overline{T}_{p(H_{\frac{1}{3}})}$ and $\overline{T}_{p(\frac{1}{3})}$ is well established. The empirical ratio $\frac{T_s}{\overline{T}}$ of 1.2 (Kitano et al., 2002) or 0.9–1.4 (Goda and Nagai, 1974; Goda, 2010) is fairly captured by both computed and estimated $\frac{\overline{T}_{p(H_{\frac{1}{3}})}}{\overline{T}_p}$.

Landwehr et al. (1979), and McMahon and Srikanthan (1982) explored the influence of serial dependence on at-site frequency analysis and observed that they caused irrelevant bias in quantile appraisals. Hosking and Wallis (1997) reasoned that a small amount of serial dependence in annual data series has little impact on the quality of quantile estimates. They also considered that when the data are generated from a physical process that can produce anomalies, then a distribution that is a close fit to the current observed data will not ensure the same to the future data. So, it is desirable to use a robust approach based on a distribution that will yield acceptably precise quantile approximations even when the true at-site frequency distributions differ from the fitted frequency distributions. The purpose of this study is to evaluate the accuracy of the parametric relations, derived from the proposed distribution functions obtained by the statistical approach discussed in the following sections, to estimate various wave period statistics.