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On the stability of a class of shoreline planform models

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ABSTRACT

The evolution of beaches in response to the incident wave conditions has long attracted the attention of researchers and engineers. A popular mathematical model describing the change in the position of a single height contour on the coastline assumes that the beach profile is stable and the plan shape evolves due to wave-driven long-shore transport. Extensions of this model include more contours and allow for beach profile alteration through cross-shore transport of sediment. Despite this advantage, models with multiple contours remain relatively underused. In this paper we examine the stability of this class of model for the cases of one to three contours. Unstable modes may exist when there is more than one contour. These include short waves whose growth rate is strongly dependent upon wavenumber. For the case of three contours an additional long wave instability is possible. A necessary, but not sufficient, condition for instability is found. It requires a reversal of transport direction amongst the contours. The existence of these instabilities provides a possible explanation for the difficulties found in implementing computational multi-line models, particularly where structures alter the natural longshore transport rates so they satisfy, locally, the condition for instability.

1. Introduction

The problem of beach stability, particularly with respect to shoreline response to the construction of coastal defences, has long been a familiar question in coastal research. Early work can be traced back to the 1950s, with continuing research since that time. Interest in this problem stems from the observation of instabilities in the natural environment. It is also germane to the problem of formulating numerical models that describe the time evolution of the shoreline. Whilst it was common to commission laboratory studies to test the performance of proposed coastal structures the era of cheap computing has meant that computational modelling has become an important design tool that doesn't suffer from the scaling problems inherent in modelling sediment transport in laboratory models. However, computational modelling has brought new challenges, particularly in view of the need to have reassurance that results from a computer model are reliable, stable and robust. One source of checks on the performance of a computer model is analytical solutions. As a general rule such solutions are available only for simplified situations or a restricted range of conditions. Nevertheless, analytical solutions remain one of the best forms of basic computer model testing. Checking the performance of computer models in more complex situations requires comparison against laboratory or field experiments. Current design practice typically relies on several if not all of the above approaches. In this paper we focus on analytical methods and the characteristic behaviour of beach models when some of the simplifications adopted for analytical solutions are relaxed. Whilst this might at first appear to be an exercise of solely academic interest the results have relevance for a class of computational models known as line models. The simplest of these models is the 1-line model, which describes the movement of a single contour line in response to incoming waves and sources and/or interruptions of sediment supply along the beach. This model has a fairly restrictive assumption that the beach profile remains unchanged. It was only natural that researchers wished to remove this restriction, so models describing more contour lines were developed so as to account for changes in beach profile shape. Nevertheless, these 'better' models which might have been expected to provide more realistic results, have not entered mainstream use. One reason seems to have been difficulties encountered in achieving solutions that converge. One potential reason for this is numerical instability, in which small fluctuations in the solution can grow rapidly, overwhelming the features of engineering interest, and rendering the computed solutions meaningless. In this paper we analyse the stability properties of the 1, 2 and 3 line models to investigate whether such instabilities might be an inherent property of the systems of equations, thereby rendering the computational approach rather problematic in certain situations. In Section 2 the 1-line model, which is widely known in the coastal engineering literature, is discussed briefly. In Sections 3 and 3.1 the stability of the governing equations to small perturbations in the beach position is assessed. Conclusions from the assessment are presented in Section 3.2.

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2. The 1-line model and beyond

Pelnard-Considère (1956) undertook laboratory experiments and proposed a simplified analytical model to describe the long-shore sediment transport driven by waves. Assuming constant, uniform wave conditions and that the angle between wave crests and the shoreline contour is small, the equation for the position of the shoreline (or a single depth contour), $y(x, t)$, from a fixed datum line takes the form of a linear diffusion equation with constant coefficient and has gained the epithet of the 'one-line equation':

$$\frac{\partial y}{\partial t} = K\frac{\partial^2 y}{\partial x^2}. \tag{1}$$

In Eq. (1) y is the distance of the reference contour from a datum line (usually taken to be the x-axis), t is time, and K is a diffusion coefficient that represents the factors affecting the rate at which sediment is transported along the shoreline, and may be written as $2Q_0/D$ where D is the height of the active profile and Q_0 is a nominal sediment transport rate that depends upon wave height and sediment characteristics. The physical transport rate is equal to the product of Q_0 and a function of wave angle, determined empirically to be $\sin(2\alpha_b)$ where α_b is the angle between the breaking wave crests and the local shoreline contour (US Army Corps of Engineers, 2002). Specifically,

$$K = \frac{2}{D}\frac{\kappa\rho H_{sb}^2 C_{gb}}{16(\rho_s-\rho)(1-p_s)} \tag{2}$$

where p_s is the porosity of the beach material, ρ_s is the density of the beach sediment (kg/m^3), ρ is the density of seawater (kg/m^3), H_{sb} is the significant wave height at breaking (m), c_{gb} is the wave group velocity at breaking (m/s), and κ is a dimensionless coefficient which is a function of the particle size and has a recommended value of approximately 0.4 for sandy beaches.

Some of the first analytical solutions to this equation for cases of coastal engineering interest were presented by Grijm (1960), under the assumption of constant uniform waves. This assumption can be relaxed, but complicates the analysis considerably. Larson et al. (1997) presented an equation for constant but non-uniform wave conditions corresponding to Eq. (1), whilst Reeve (2006), Zacharioudaki and Reeve (2008), Walton and Dean (2011) and Valsamidis et al. (2013) have presented solutions for cases where wave conditions are uniform but time varying. The development of analytical models not only answers a pedagogical need but also broadens the range of conditions for testing computational models. Whilst some of the methods required to derive analytical solutions can appear difficult and arcane, the solutions themselves have several attractive properties. First, they can usually be evaluated in a straightforward and efficient manner. Second, no time stepping is required as the solution simply needs to be evaluated at a selected time, rather than approached iteratively through multiple time steps. Finally, analytical solutions do not suffer from numerical stability in the same way that computational models can. The need to make simplifying assumptions does restrict analytical methods and for many practical applications numerical models are preferred, particularly as they can be linked to other models that can predict, for example, nearshore wave transformation processes.

In practical applications, the 1-line model is solved using a time marching numerical solution procedure to solve the continuity, sediment transport and wave angle equations simultaneously, commonly relaxing the 'small angle approximation' made to assist in deriving analytical solutions. Wang and Le Méhauté (1980) showed that the expression for sediment transport can, under the assumption of parallel depth contours, be cast in terms of deepwater wave conditions. Instabilities can occur for certain values of offshore wave angle, although the equation governing shoreline evolution no longer takes the form in Eq. (1). This idea was used by Ashton et al. (2001) to develop a numerical model that provided an explanation for the unusual coastal spit formations found in the Azov Sea, and by Falques (2003) in an analysis of the method of estimating the diffusion coefficient.

The 1-line model has proved remarkably robust, and over the past decade or so has been used extensively, being an element in current design guidelines (eg. US Army Corps, 2002). The use of the 1-line model in more complicated situations has driven the development and expansion of a simple morphological equation (Eq. (1)) into computational prediction suites. For example, such modelling suites include elements of wave prediction, nearshore wave transformation, modifications to allow for wave diffraction, longshore variations in wave angle and height, and variations in beach slope (eg Hanson and Kraus, 1989, Dabees & Kamphuis, 1999).

One criticism of the 1-line formulation is that it is not able to deal explicitly with cross-shore sediment transport and hence changes in the cross-shore beach profile. Changes in beach slope and convexity that arise from the construction of coastal defences require something more than the 1-line model. Bakker (1969) proposed a 2-line theory which predicted the simultaneous evolution of two distinct contour lines and hence changes in beach slope. Perlin and Dean (1983) subsequently presented the theory for an N-line model together with associated numerical solution methods. With a few notable exceptions (Dabees and Kamphuis, 2001; Hanson and Larson, 2001; Shibutani et al., 2009), the N-line formulation has not found widespread application. The reasons for this are not entirely clear, although there have been some reports discussing the difficulty of obtaining consistent results in practical applications. For example, Hulsbergen et al. (1976) performed laboratory experiments and found that the 2-line model worked well for a long, straight beach with a simple pattern of longshore currents but was not as accurate for more complicated situations; whilst Hanson and Larson (2001) commented upon instances of numerical difficulties in certain applications.

3. The 2-line model

The stability of an equation, or system of equations, can be investigated by analysing the propensity of small perturbations to grow or decay over time. Perturbations are typically taken to have a sinusoidal form. Seeking wave solutions to Eq. (1) of the form $y(x,t) = Y exp\{i(kx - \omega t)\}$ leads to the requirement that $\omega = -iKk^2$, where ω is the angular frequency of the perturbations, i is the imaginary unit and k is the wave number of the perturbations in the position of the shoreline contour. This requirement indicates that any wave solutions will be damped over time. It is a matter of convention as to over what ranges the wavenumber and frequency are allowed to vary. Here, we take k as being non-negative whilst ω may take on positive or negative values, depending upon which direction the perturbation propagates along the shoreline.

Following Bakker (1969), a 2-line model can be formulated on the argument that the beach profile can be divided into two parts, a shallow part that extends offshore to a depth D_1 and a deeper part that extends out to the depth of closure, written as $D_1 + D_2$ (see Fig. 1).

Again the concept of an equilibrium profile is used but in this case it is defined by the position of two contours together with the position of the depth of closure. In Fig. 1, y_1 is the distance of a chosen contour (typically the equilibrium sea level), from a fixed datum line landward of the shoreline. This datum line is usually taken as the 'x-axis'. A second contour, a distance w from the x-axis at equilibrium, is denoted by y_2. When the profile is in equilibrium, both y_1 and y_2 are zero. When either of the contours is not in their equilibrium position a cross-shore sediment transport is inferred. The cross-shore transport of sediment between the two contours must satisfy continuity and is assumed, following Bakker (1969), to be in a sense that relaxes the beach profile towards its equilibrium form. Hence, it may be written as $q_y = C_y(y_1 - y_2)$ where C_y is a dimensional transport coefficient. It may be noted that even though y_1 is chosen to be the contour of the equilibrium