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A 2DH nonlinear Boussinesq-type wave model of improved dispersion, shoaling, and wave generation characteristics

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ABSTRACT

A modified Boussinesq-type model is derived to account for the propagation of either regular or irregular waves in two horizontal dimensions. An improvement of the dispersion and shoaling characteristics of the model is obtained by optimizing the coefficients of each term in the momentum equation, expanding in this way its applicability in very deep waters and thus overcoming a shortcoming of most models of the same type. The values of the coefficients are obtained by an inverse method in such a way as to satisfy exactly the dispersion relation in terms of both first and second-order analyses matching in parallel the associated shoaling gradient. Furthermore a physically more sound way to approach the evaluation of wave number in irregular wave fields is proposed. A modification of the wave generator boundary condition is also introduced in order to correctly simulate the phase celerity of each input wave component. The modified model is applied to simulate the propagation of breaking and non-breaking, regular and irregular, long and short crested waves in both one and two horizontal dimensions, in a variety of bottom profiles, such as of constant depth, mild slope, and in the presence of submerged obstacles. The simulations are compared with experimental data and analytical results, indicating very good agreement in most cases.

1. Introduction

Wind waves are one of the most important driving factors in the coastal environment. Understanding wave hydrodynamics and the processes induced by them is important for designing marine structures and for managing of the coastal zone. Numerous researchers have contributed to the development of analytical theories and numerical models aiming at simulating wave propagation and describing wave transformation due to various phenomena such as wave shoaling, diffraction, refraction, and breaking. Wave models based on the original Boussinesq equation have proven to be quite accurate, especially when applied to relatively shallow water regions (Peregrine, 1967). Further developments, loosely named Boussinesq-type models, can incorporate highly nonlinear wave characteristics partially disregarding the original Boussinesq assumptions and simulate fully dispersive properties. Nevertheless all of the above models normally lose some of their efficiency when simulating wave propagation in deep waters. Thus each of these models has restrictions and specific applicability limits, e.g. $kd \leq 1$, where d is water depth and k wave number. This is considered a serious drawback since wind waves are normally generated and start their travel in deep waters and such models cannot, therefore, cover adequately the whole domain stretching from deep waters to the shore. A further complication arises when dealing with real life sea states, that are viewed as composed of a variety of individual waves with different characteristics.

Various attempts have been made to extend and practically eliminate water depth limitation, e.g. Madsen et al. (1991), Nwogu (1993), Wei et al. (1995), Zou (1999), extending the water depth limit up to $kd \approx 3$, i.e. to the deep water boundary. Madsen and Schäffer (1998), referred to as MS98 hereafter, derived two models, in terms of depth averaged velocity and in terms of particle velocity at an arbitrary location along the vertical, achieving acceptable accuracy for depths up to $kd = 6$. More recent efforts offered a wider depth range to Boussinesq-type models, providing accurate linear dispersion and shoaling characteristics for $kd > 3$. Madsen et al. (2002) presented a model having as a starting point the exact solution to the Laplace equation, where they replaced the infinite series operators by finite series approximations involving up to fifth-derivative order operators. By manipulating the finite series to incorporate Padé approximants the linear and nonlinear characteristics became very accurate up to $kd = 40$. A different approach was implemented by Li (2008). In that procedure a hyperbolic cosine function of the Stokes first-order solution was used as a first approximation, instead of the velocity potential function at the bottom, deriving a fully dispersive model. A new formulation of Boussinesq model was introduced by Karambas and Memos (2009), referred to as KM09 hereafter. This model offers significant advantages due to the small number of terms involved in both mass and momentum

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conservation equations and simulates accurately the propagation of fully dispersive water waves.

Despite all of this progress several problems remain concerning the nonlinear wave characteristics, the irregular wave propagation and the consequent complexity of the system of partial differential equations, which in many models include a significant number of terms. Recently a preliminary proposal was presented by Chondros and Memos (2012), referred to as CM12 hereafter, to define new coefficients in MS98 equations in order to improve its dispersion characteristics in any depth. These coefficients, that govern the linear and nonlinear dispersion and appear in the momentum equation, were allowed to be time and space dependent in contrast with all past studies of Boussinesq-type models, where they were assumed constants. Hence improvement in the wave dispersion and shoaling characteristics at first and second-order analysis was achieved. This technique may also be useful for direct application in other Boussinesq models as well. In this paper the following three steps are taken to briefly present and mainly to improve the CM12 model. Firstly, the derivation of the coefficients that govern the linear shoaling characteristics of the model is recapitulated, secondly, a more physically based approach to handle the evaluation of wave number k in irregular waves is proposed and finally a modification in wave generator is performed in such a way to simulate accurately the linear dispersion of generating waves in any water depth. The following chapters present the derivation of continuity and momentum equations following MS98, the calculation of the new set of coefficients, the modification of the numerical wave generator to be applicable to any water depth, and finally the calibration and verification of the modified model by comparing its results with experimental data and with mathematical theories. To this effect tests are used with breaking and non-breaking, regular and irregular, long and short crested waves propagating over a mild slope, over a reef trapezoidal bar and over an elliptic shoal in both 1HD and 2HD configurations.

2. Theoretical formulation

2.1. Equations by Madsen and Schäffer (1998)

The continuity equation in a wave of two horizontal dimensions (x, y) is written as

$$\frac{\partial \zeta}{\partial t} + \nabla((d + \varepsilon\zeta)U) = 0 \tag{1}$$

where U is the depth averaged velocity, ζ is the surface elevation, and ε the nonlinearity parameter equal to H/d (where H is the local wave height). Assuming a rather strong nonlinearity, i.e. $\varepsilon = O(\mu)$ with μ is the frequency dispersion parameter equal to d/L (where L is the local wavelength), and considering a mildly sloping bottom where only the first derivatives of d are included, the momentum equation in two horizontal dimensions is given, following MS98:

$$\begin{aligned}\frac{\partial U}{\partial t} + \nabla\zeta + \frac{1}{2}\varepsilon\nabla\left(U^2\right) + \mu^2\left(\Lambda_{20}^{\mathrm{II}} + \varepsilon\Lambda_{21}^{\mathrm{II}} + \varepsilon^2\Lambda_{22}^{\mathrm{II}} + \varepsilon^3\Lambda_{23}^{\mathrm{II}}\right) \\ + \mu^4\left(\Lambda_{40}^{\mathrm{II}} + \varepsilon\Lambda_{41}^{\mathrm{II}}\right) + O\left(\mu^6, \varepsilon^2\mu^4\right) = 0\end{aligned} \tag{2}$$

where for the Λ^{II} terms the reader is referred to the original paper MS98.

In order to improve the linear and nonlinear dispersion and the linear shoaling characteristics, MS98 introduced four free parameters $(\alpha_1, \beta_1, \alpha_2, \beta_2)$ and applied a three-step operational procedure on Eq. (2). They then obtained the following new alternative higher-order momentum equation:

$$\begin{aligned}\frac{\partial U}{\partial t} + \nabla\zeta + \frac{1}{2}\varepsilon\nabla\left(U^2\right) + \mu^2\left(\Lambda_{20}^{\mathrm{III}} + \varepsilon\Lambda_{21}^{\mathrm{III}} + \varepsilon^2\Lambda_{22}^{\mathrm{III}} + \varepsilon^3\Lambda_{23}^{\mathrm{III}}\right) \\ + \mu^4\left(\Lambda_{40}^{\mathrm{III}} + \varepsilon\Lambda_{41}^{\mathrm{III}}\right) + O\left(\mu^6, \varepsilon^2\mu^4\right) = 0\end{aligned} \tag{3}$$

The Λ^{II} terms are reproduced from MS98 in the appendix for ease of reference.

The set (α_1, β_1) governs the dispersion relation, while the set (α_2, β_2) can be used to optimize the linear shoaling gradient. MS98 tried to optimize the linear dispersion characteristics by selecting the set of (α_1, β_1) to converge close to the respective target result of Stokes linear wave:

$$\left(\frac{\omega^2}{k^2 d}\right)^{Stokes} = \frac{\tanh(\kappa)}{\kappa} \tag{4}$$

where $\kappa = kd$, k the wave number and ω is the angular frequency.

For values $(\alpha_1, \beta_1) = (1/9, 1/945)$ the accuracy of linear dispersion is seen to be excellent for kd up to 6, which is twice the traditional deep-water limit. They also optimized the linear shoaling characteristics by selecting the set of (α_2, β_2) to converge to the corresponding target result of Stokes linear wave theory. To do this they used the notion of linear shoaling gradient γ_0 introduced previously by Madsen and Sørensen (1992) as another important quantity able to measure the applicability of Boussinesq equations. This gradient is a function of the local κ and is defined by:

$$\frac{A_x}{A} = -\frac{d_x}{d}\gamma_0 \tag{5}$$

where A is the local wave amplitude and the subscript denotes partial derivative.

The Stokes result is then written as

$$\gamma_0^{Stokes} = \frac{2\kappa\sinh 2\kappa + 2\kappa^2(1 - \cosh 2\kappa)}{(2\kappa + \sinh 2\kappa)^2} \tag{6}$$

For values $(\alpha_2, \beta_2) = (0.146488, 0.00798359)$ the accuracy they achieved was excellent for kd up to 6 as with the previous set of coefficients.