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Optimization of non-hydrostatic Euler model for water waves

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ABSTRACT

The distribution of variables in the water column controls the dispersion properties of non-hydrostatic models. Because solving the Poisson equation is the most time consuming part of a non-hydrostatic model, it is highly desirable to reduce the number of unknowns in the water column by placing them at optimal locations. The paper presents the analytical dispersion relationship of a non-hydrostatic Euler model for water waves. The phase speed of linear waves simulated by the semi-discretized Euler model can be expressed as a rational polynomial function of the dimensionless water depth, kh, and the thicknesses of layers encompassing the water column become optimizable parameters in this function. The dispersion error is obtained by comparing this phase speed against the exact solution based on the linear wave theory. It is shown that for a given dispersion error (e.g. 1%), the range of kh can be extended if the layer thicknesses are optimally selected. The optimal two- and three-layer distributions for the aforementioned Euler model are provided. The Euler model with the optimized layer distribution shows good linear dispersion properties up to $kh \approx 9$ with two layers, and $kh \approx 49.5$ with three layers. The derived phase speed was tested against both numerical and exact solutions of standing waves for various cases. Excellent agreement was achieved. The model was also tested using the fifth-order Stokes theory for nonlinear standing waves. The phase speed of nonlinear waves follows a similar trend of the derived phase speed although it deviates proportionally with the increase of wave steepness ka_0. Thus, the optimal layer distribution can also be applied to nonlinear waves within a range of ka_0 and kh. The optimization method is applicable to other non-hydrostatic Euler models for water waves.

1. Introduction

In physical oceanography, and coastal and ocean engineering, an accurate and efficient prediction of surface waves is of paramount importance. In the past several decades with the rapid advances in computational technology, substantial efforts have been devoted to developing numerical models for simulating the propagation and evolution of water waves from deep water to the shoreline.

A widely used type of models is based on depth-averaged equations, such as Boussinesq-type models. The conventional Boussinesq models (e.g. Peregrine 1967) have the drawback of weak dispersion and weak nonlinearity, or a limited range of applicability. By introducing a reference velocity at an arbitrary vertical location as the velocity variable and choosing the location to match the Padé [2, 2] expansion of the exact linear dispersion relationship, Nwogu (1993) extended the applicability of the model to $kh \approx 3$, here k is the wave number, and h is the water depth. The dimensionless wave number kh serves as a dispersion criterion.

Many efforts have been put in to enhance the deep-water accuracy of the depth-integrated approach by developing high-order Boussinesq-type models. By utilizing a quartic polynomial approximation for the vertical profile of velocity field, Gobbi and Kirby (1999) developed a model with excellent linear dispersion properties up to $kh \approx 6$. The models of Madsen and Schäffer (1998), and Chen et al. (1998) also showed good linear dispersive wave properties up to $kh \approx 6$. Madsen et al. (2002) further improved the Boussinesq-type model and made it applicable to extremely deep-water waves with $kh \approx 40$. Agnon et al. (1999) presented a new procedure to achieve good nonlinear dispersion properties up to $kh \approx 6$.

Lynett and Liu (2004a and 2004b) introduced another approach to improving the dispersion accuracy of Boussinesq-type models. Instead of using a high-order approximation for the vertical profile of the flow field, they used N independent quadratic polynomial approximations for the velocity profiles. These polynomial approximations match at the interfaces that divide the water column into N layers. Good dispersion accuracy up to $kh \approx 8$ was achieved with an optimized two-layer model. Chazel et al. (2009) combined the approach in Madsen et al. (2002) and the approach in Lynett and Liu (2004a), and developed an

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efficient model with less complexity. This model exhibits good linear and nonlinear dispersion properties to $kh \approx 10$.

Another widely used type of models is based on the Navier–Stokes equations (NSE) or the Euler equations of fluid motion. The momentum equations are solved at every layer for this type of models, thus the computational cost is higher compared with the Boussinesq type models. To capture the rapid changing free surface and to apply the pressure boundary condition precisely on the free surface of the waves, Navier–Stokes models involve the non-hydrostatic pressure on the top layer, and solve this problem by methods of: (1) employing edged-based grid systems (e.g. Lin and Li, 2002; Zijlema and Stelling, 2005, 2008; Zijlema et al., 2011; Chen et al., 2011; Ma et al., 2012), (2) using the integration method (e.g. Yuan and Wu, 2004), (3) implementing interpolation approaches (e.g. Walters, 2005; Choi and Wu, 2006; Young et al., 2007), and (4) embedding Boussinesq-type like equation into the Navier–Stokes model (e.g. Wu et al., 2010).

To improve the model dispersion properties, non-uniform layers have been used. Finer vertical resolution is utilized near the free surface for capturing the rapid velocity and pressure variation while coarse resolution is used at the bottom due to the mild change of velocity and pressure in this region. Yuan and Wu (2006) presented a so-called top-down resolving (TDR) technique to determine a set of suitable thickness for layers. The basic concept of this technique is to keep a fine resolution from the top layers down to the one just above the bottom layer, and use a coarse resolution for the bottom layer. With this TDR method, their model can achieve the same dispersion accuracy using 5 non-uniform layers rather than using 10 uniform layers for waves with $kh = 4$.

Young and Wu (2009) provided the top-layer thickness for their two-layer non-hydrostatic model. For $kh = 3.14$, the top layer takes 50% of the water depth; for $kh = 6$, the top layer takes 39.3% of the water depth, and for $kh = 15$, the top layer takes 15.7% of the water depth. However, the dependence of the top layer thickness on kh values hinders the model application in resolving highly dispersive random waves. Young and Wu (2010) proposed an adapted top-layer control (ATLC) method that eliminates the model layer thickness dependence on kh. The basic concept is that the finer resolution of the top layer is given by $\Delta\sigma_N = 0.5 / (N - 1)$, where $\Delta\sigma_N$ is the top layer size, and N is the number of layers. The size of the rest layers exponentially stretches to the bottom layer. The top layer size is set as 50%, 25% and 12.5% for two-, three-, and five-layer models, respectively. For a given tolerance phase error of 1%, the two-, three-, and five-layer models are capable of resolving linear wave dispersion up to $kh = 3.14$, 6.28 and 15.7, respectively.

Zijlema and Stelling (2005, 2008) and Zijlema et al. (2011) proposed a non-hydrostatic, free-surface flow model, SWASH (an acronym of Simulating WAves till SHore) for simulating wave propagation from deep water to the surf zone, using a semi-implicit, staggered finite volume method. The adoption of the Keller-box scheme in this model enabled the pressure to be assigned at the free surface without any approximation, and also enabled good dispersion properties at very low vertical resolution (two or three layers). Using two uniform layers, for a 1% tolerance phase error, SWASH achieves good linear dispersion properties up to $kh \approx 7$ and 3 for standing waves and progressive waves, respectively. Using three uniform layers, for a 1% tolerance phase error, SWASH achieves good linear dispersion properties up to $kh \approx 7$ for progressive waves. Zijlema et al. (2011) suggested that adjusting layer thicknesses could significantly improve wave dispersion accuracy. With layer thickness tuned as 10%, 20%, and 70% of the total water depth, their three-layer model is able to simulate waves with $kh \approx 15.7$ with a relatively small phase error.

Bai and Cheung (2013) provide theoretical dispersion relationships for an N-layer Boussinesq-type system. The derived dispersion relationships follow a $[2N - 2, 2N]$ Padé expansion. The accumulative dispersion error is computed over $0.01 \leq kh \leq 6$ for covering a wide spectrum of ocean waves from deep water to shoreline. The layer arrangement appears as free parameter and can be adjusted for minimizing the accumulative dispersion error, and optimizing the proposed model. The two-layer model with the optimal layer arrangement of 50%–50% of the water depth gives a minimum error of 3.445%. The three-layer model with the optimal layer arrangement of 29%–32%–39% of the water depth gives a minimum error of 3.09%.

Similar to Boussinesq-type models in which the location of the horizontal velocity in the water column controls the dispersion properties, the vertical distribution of the variables in non-hydrostatic free surface flow models dictates the model dispersion accuracy. Because solving the Poisson equation is the most time consuming part of a non-hydrostatic model, it is highly desirable to reduce the number of unknowns in the water column by placing them at optimal locations. The objective of our study is to derive the analytical dispersion relationship of a non-hydrostatic free surface flow model and optimize it for extremely dispersive and highly nonlinear waves with only two or three layers. Although the optimization method is applicable to all non-hydrostatic models, for the demonstration purpose, the present work chose a Euler model that uses the algorithm and numerical scheme presented in Zijlema and Stelling (2005), except using the finite difference method in a sigma-coordinate. In this paper, we will present: (1) the derivation of the phase speed of linear waves simulated by the semi-discretized non-hydrostatic Euler model, and (2) the optimal two- and three-layer distributions. For a 1% tolerance dispersion error, the optimal two-layer distribution from bottom to top is 67% and 33% of the total water depth and the Euler model is capable of simulating linear waves up to $kh = 9$. For a 1% tolerance dispersion error, the optimal three-layer distribution from bottom to top is 68%, 26.5% and 5.5% of the total water depth and the Euler model is capable of simulating extremely dispersive waves up to $kh = 49.5$.

Furthermore, nonlinear standing wave tests have been performed to examine the effect of nonlinearity. Kirby and Dalrymple (1986) proposed a nonlinear dispersion relation, which effectively approximated the dispersion characteristics of waves propagating from deep to shallow waters. The nonlinear effect was scaled by wave steepness ka_0, and the phase speed became a function of kh and ka_0. In this paper, numerical experiments with different ka_0 are conducted. A fifth-order Stokes-type exact solution for standing waves (Sobey, 2009) is utilized to test our model. We have found that the phase speed of nonlinear waves follows the analytically derived linear phase speed, although the discrepancy rises as ka_0 increases. Therefore the optimal layer distribution can also be applied to the nonlinear wave simulation.

This paper is organized as follows: Section 2 briefly describes the governing equations and the numerical schemes. Section 3 presents the numerical results in comparison with the exact solution to demonstrate the performance of the model. In Section 4, we derive the analytical dispersion relationship of this discretized system of Euler equations followed by the determination of the optimal two- and three-layer distributions for extremely dispersive waves. Section 5 is focused on the effect of wave nonlinearity on the model performance with the optimal layer distributions. Finally, Section 6 summarizes the findings and presents the conclusions.

2. The Euler model

2.1. Governing equations

The propagation of fully dispersive, nonlinear water waves is governed by the continuity equation and the incompressible Euler equations, which are written as

$$\frac{\partial u}{\partial x^*} + \frac{\partial w}{\partial z^*} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t^*} + u\frac{\partial u}{\partial x^*} + w\frac{\partial u}{\partial z^*} + \frac{\partial p}{\partial x^*} = 0 \tag{2}$$