Coastal Engineering 83 (2014) 93–107

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

Simulation of nearshore wave processes by a depth-integrated non-hydrostatic finite element model

Zhangping Wei *, Yafei Jia

National Center for Computational Hydroscience & Engineering, The University of Mississippi, University, MS 38677, USA

ARTICLE INFO

Article history:
Received 6 July 2013
Received in revised form 27 September 2013
Accepted 3 October 2013
Available online 30 October 2013

Keywords:
Non-hydrostatic
Wave breaking
Wave run-up
Depth-integrated
Momentum conservation
CCHE2D

ABSTRACT

This paper presents CCHE2D-NHWAVE, a depth-integrated non-hydrostatic finite element model for simulating nearshore wave processes. The governing equations are a depth-integrated vertical momentum equation and the shallow water equations including extra non-hydrostatic pressure terms, which enable the model to simulate relatively short wave motions, where both frequency dispersion and nonlinear effects play important roles. A special type of finite element method, which was previously developed for a well-validated depth-integrated free surface flow model CCHE2D, is used to solve the governing equations on a partially staggered grid using a pressure projection method. To resolve discontinuous flows, involving breaking waves and hydraulic jumps, a momentum conservation advection scheme is developed based on the partially staggered grid. In addition, a simple and efficient wetting and drying algorithm is implemented to deal with the moving shoreline. The model is first verified by analytical solutions, and then validated by a series of laboratory experiments. The comparison shows that the developed wave model without the use of any empirical parameters is capable of accurately simulating a wide range of nearshore wave processes, including propagation, breaking, and run-up of nonlinear dispersive waves and transformation and inundation of tsunami waves.

1. Introduction

Nearshore wave processes are the major forces that dominate coastal sediment transport, shape coastal morphology, threaten coastal infrastructures, and cause flooding. In recent years, considerable efforts have been made to develop accurate and efficient wave models for predicting propagation, breaking, and run-up of nonlinear dispersive waves in the coastal zone. Numerical models built on the well-established Boussinesq-type equations have been widely used to simulate wave motions. The basic idea of Boussinesq-type equations is the elimination of the vertical coordinate, while accounting for some influences of the vertical acceleration (Dingemans, 1997). The classical Boussinesq equations of Peregrine (1967) only consider weak nonlinearity and weak dispersion, and therefore, his formulation is only applicable to $kh < 0.75$ (where k is the wave number and h is the still water depth) in practice (Madsen et al., 2002). To reduce these constraints, several enhanced and higher-order Boussinesq-type equations (e.g., Agnon et al., 1999; Gobbi et al., 2000; Madsen and Sørensen, 1992; Madsen et al., 2002; Nwogu, 1993; Wei and Kirby, 1995) have been developed. However, these higher-order equations usually involve complex numerical discretization and expensive computation. In order to apply the Boussinesq-type models for simulating nearshore processes, wave breaking has to be treated carefully. Since Boussinesq-type equations are depth-integrated formulations in nature, they cannot describe the overturning of the free surface. In addition, the classical Boussinesq equations were derived under the assumption of an irrotational and inviscid flow (Peregrine, 1967), so they cannot directly account for energy dissipation caused by wave breaking. Several semi-empirical approaches have been developed to address these issues (e.g., Schäffer et al., 1993; Veeramony and Svendsen, 2000; Zelt, 1991). In general, these approaches have to add an extra dissipation term in the momentum equation, and provide predefined criteria for the onset and cessation of wave breaking and energy dissipation rate by calibration using laboratory experiments. In this regard, the reliability of those tunable parameters is likely dependent on the similarity between the considered case and the experiment, and the accuracy of the experimental data.

With the rapid development of computer hardware and numerical methods, it becomes feasible to simulate wave propagation by directly solving the three-dimensional (3D) Reynolds-averaged Navier–Stokes (RANS) equations. Using this approach, the vertical flow structure can be simulated together with the horizontal flow field. With the aid of free surface tracking methods, such as the Marker-and-Cell method (Harlow and Welch, 1965), the Volume-of-Fluid method (Hirt and Nichols, 1981) and so on, solution of the 3D RANS equations has been successfully applied to simulate wave breaking and run-up, as reported in the literature; see Lin and Liu

* Corresponding author. Tel.: +1 662 915 7788; fax: +1 662 915 7796.
E-mail addresses: zpwei@ncche.olemiss.edu (Z. Wei), jia@ncche.olemiss.edu (Y. Jia).

0378-3839/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.coastaleng.2013.10.002

(1998), Lin et al. (1999), and Christensen and Deigaard (2001), among others. However, the high computational expense and strict stability requirement of these free surface tracking methods make applications to large-scale domains impracticable.

Another approach, the so-called non-hydrostatic method for simulating water waves still solves the RANS equations, but it explicitly includes the non-hydrostatic pressure to account for the vertical acceleration of flows (Casulli and Stelling, 1998; Stansby and Zhou, 1998). Compared with numerical wave models built on the 3D RANS equations with free surface tracking methods, the non-hydrostatic model usually solves the free surface motion with a single value function in terms of horizontal coordinates and time, and it therefore requires a much lower vertical resolution than that of the free surface tracking methods. This property makes the non-hydrostatic model a favorable choice to large-scale applications in coastal engineering. An important issue that determines the efficiency and accuracy of the non-hydrostatic model is the free surface pressure boundary condition. In earlier non-hydrostatic model developments, due to a staggered grid configuration in the vertical direction, the non-hydrostatic pressure is specified at the cell center, and the hydrostatic pressure assumption is made for the free surface layer (e.g., Casulli, 1999; Casulli and Stelling, 1998; Casulli and Zanolli, 2002; Chen, 2003; Li and Fleming, 2001; Namin et al., 2001). This approximation ignores the non-hydrostatic pressure effect at the free surface. As a result, a relatively large number of vertical layers (i.e., 10–20 layers) are required to resolve the wave dispersion to an acceptable level of accuracy.

Stelling and Zijlema (2003) proposed an edge-based compact difference scheme to approximate the vertical gradient of the non-hydrostatic pressure located at the interface between vertical layers. In this way, the zero pressure boundary condition can be specified at the free surface precisely. It turns out that their two-layer non-hydrostatic model shows similar linear dispersion characteristics as the extended Boussinesq-type models of Madsen and Sørensen (1992) and Nwogu (1993). Later on, they implemented an efficient and stable solver for the non-hydrostatic pressure (Zijlema and Stelling, 2005) and released an operational public domain code: SWASH (Zijlema et al., 2011). To date, their approach has been adopted in other models for simulating wave motions (e.g., Ai et al., 2010; Cui et al., 2012; Ma et al., 2012; Walters, 2005; Wei and Jia, 2013a; Yamazaki et al., 2008).

Similar to Boussinesq-type models, non-hydrostatic models simulate the lumped effects of wave breaking rather than the detailed process. So far, the techniques for modeling wave breaking used by non-hydrostatic models are based on the physical principle applied in open channel hydraulics. It has been observed that the nonlinear shallow water equations can appropriately handle discontinuous flows (e.g., hydraulic jumps and bores) with steep gradients, if the momentum is conserved at the discretized level (Stelling and Duinmeijer, 2003). Consequently, non-hydrostatic models with this property are able to track the actual location of wave breaking and compute the associated energy dissipation correctly without the application of any predefined criteria and empirical parameters as used by Boussinesq-type models (Zijlema and Stelling, 2008). To achieve momentum conservation in the formulation, two strategies have been utilized. Stelling and Duinmeijer (2003) developed a momentum conservation scheme on the basis of the fully staggered grid, and their scheme has been employed to handle wave breaking when the non-conservation form of governing equations is considered (Yamazaki et al., 2008; Zijlema et al., 2011). The other way is to solve the conservation form of governing equations directly, since momentum conservation is automatically taken into account (Ai and Jin, 2012; Ma et al., 2012; Zijlema and Stelling, 2008). When it comes to the determination of energy dissipation due to wave breaking in the scope of non-hydrostatic formulations, it is straightforward to make use of the eddy viscosity since it is a part of the RANS equations. Jacobs (2010) and Ma et al. (2012) adopted the subgrid-scale model of Smagorinsky (1963) into non-hydrostatic models to compute energy dissipation induced by wave breaking and subgrid turbulence. The basic idea behind their implementations is that the grid size involved with the simulation of surface waves is usually smaller than the typical water depth, and the use of the subgrid-scale model is able to prevent a chaotic current field generated by wave breaking (Chen et al., 1999). With respect to the turbulence models of SWASH, Zijlema et al. (2011) implemented the Prandtl mixing length model to calculate the horizontal eddy viscosity, and Smit et al. (2013) further added the subgrid-scale model of Smagorinsky (1963) and a $k-\varepsilon$ model to account for the horizontal and vertical exchange of momentum, respectively. A major concern recently addressed in the development of non-hydrostatic models is related to the model efficiency when it is applied to the surf zone, in which energetic wave breaking process requires the model with a relatively high vertical resolution (i.e., 10–20 layers) to obtain accurate results, while wave processes in the rest nearshore zone can be simulated fairly well with a much lower vertical resolution (i.e., 1–3 layers). Smit et al. (2013) proposed a hydrostatic front approximation (HFA) scheme, which assumes a hydrostatic pressure distribution at the front of a breaking wave, and then the wave can rapidly transit into a bore-like shape. As a consequence, the non-hydrostatic model with a low vertical resolution can be applied to the whole nearshore zone at a significantly reduced computational cost.

In this paper, we develop a depth-integrated non-hydrostatic finite element model for simulating nearshore wave processes on the basis of our previous work (Wei and Jia, 2013a,b). The shallow water equations in the conservation form including extra non-hydrostatic pressure terms are solved together with a depth-integrated vertical momentum equation by a special finite element method based on a partially staggered grid using the pressure projection method of Stelling and Zijlema (2003). We construct a momentum conservation advection scheme on top of the partially staggered grid to ensure the model to be automatically shock-capturing, so it is able to deal with breaking waves without the use of any empirical parameters, and we also compare the numerical result with that computed with the HFA scheme. To reduce wave damping caused by numerical diffusion, a defect correction method using the advection term is also implemented. In addition, a simple and efficient wetting and drying algorithm adapted from Stelling and Duinmeijer (2003) is utilized to capture the moving shoreline. Finally, we verify and validate the model by several benchmark tests; good agreements with analytical solutions and experimental data show that the developed model is suitable for nearshore wave processes modeling.

The paper is organized as follows. The governing equations are derived in Section 2. In Section 3, the finite element method and its associated differential operators are briefly presented. And numerical formulation and solution of the governing equations are described in Section 4. In Section 5, several analytical solutions and laboratory experiments are used to verify and validate the wave model. Finally, conclusions are drawn in Section 6.

2. Governing equations

We consider a physical domain in a Cartesian coordinate system (x, y, z) that is vertically bounded by the free surface elevation $\eta(x,y,t)$ and the bed elevation $\zeta(x,y)$ as shown in Fig. 1, where t is the time, and the water depth is $H(x,y,t) = \eta(x,y,t) - \zeta(x,y)$. The incompressible 3D continuity equation and RANS equations are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{1}{\rho}\left(\frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}\right) \tag{3}$$