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Application of higher-level GN theory to some wave transformation problems

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ABSTRACT

Recently derived (Webster et al., 2011), simplified higher-level Green–Naghdi equations (GN-3, GN-5 and GN-7) are used in this work to simulate the transformation of two-dimensional, shallow-water wave problems. The spatial derivatives are discretized through a five-point difference scheme. A new algorithm is developed to solve the resulting block-pentadiagonal matrix. These high-level GN equations are then utilized to develop a numerical wave tank. A wave-maker is placed at the forcing boundary of the tank that uses the stream-function theory to generate nonlinear incident waves. The numerical wave tank is used to analyze the effects of large-amplitude waves passing over a submerged bar. A damping zone is placed near the wave-maker (up-wave side) to absorb the reflected waves from the front side of the submerged bar. Another damping zone is placed at the down-wave side of the computational domain to absorb the radiated waves. In the first test case, the front and back slopes of the bar are both mild (Luth et al., 1994). The waves that evolved over the bar are simulated by using the GN-3, GN-5 and GN-7 equations. The GN-3 equations provide time histories that compare well with the experimental data at different wave gauges, except at the ones behind the bar. The results of the GN-5 and GN-7 equations compare very well with all the experimental data considered here. In the second test case, the front and back slopes of the bar are both steep (Ohyama et al., 1995). The GN-5 equations predict the wave elevation well. In the third test case, the front and back slopes of the bar alternate, one of them being mild and the other one being steep (Zou et al., 2010). Again, the predictions of the GN-5 equations agree with the experimental data well. In all the test cases considered in this work, there are some differences between the GN-3 and GN-5 results after the crest of the bar. Numerical results obtained by the GN-5 and GN-7 equations are almost the same along the wave flume, but the GN-7 equations require more computational time. Therefore, the GN-5 results are accepted here as the converged GN theory results. The numerical validations show that the GN-5 equations can simulate the strongly nonlinear and dispersive waves observed behind the submerged bar crest satisfactorily.

1. Introduction

The Green–Naghdi (hereafter, GN) theory (Green and Naghdi, 1976; Green et al., 1974) was originally developed to analyze nonlinear free-surface flows. Because of the successful application of this method to nonlinear ship wave-making problems (Ertekin et al., 1986), the method has been subsequently applied to many nonlinear water wave problems.

The GN theory, as it is presented here, adopts a shape function representation that approximates the vertical structure of the velocity field (Demirbilek and Webster, 1992). The governing equations are the depth-integrated form of Euler's equations. Both the bottom boundary condition and the nonlinear free surface boundary conditions are satisfied exactly. Because no other assumptions and approximations are introduced, the GN equations are universally valid and can be applied to any water depth, to regular and irregular waves, and incorporate effects of varying bottom topography and vorticity (rotational). Since no restrictions are placed on the rotationality of the flow field, the GN theory can be used to study wave–current interactions, nonlinear wave–wave interactions, rip currents, infragravity wave motion, Kelvin ship waves, and highly asymmetric waves, among others. The GN theory can be applied to a full range of water depths; it is applicable to both steady and unsteady waves, including random and unidirectional and regular waves. It can deal with rapid variations in bottom topography and incorporates wave–current interactions, internal and external source/sink terms and directionally spread waves. Unlike some other shallow-water wave equations, e.g., the KdV equations, the GN equations at any level are Galilean invariant, making the solutions physical (Naghdi, 1978).

Irrotational Green–Naghdi (IGN) equations were derived by Kim et al. (2001), and deep-water version of the IGN equations has been applied to simulate ocean waves of extreme wave heights (Kim and Ertekin, 2000). The IGN equations have also been derived for finite water depth conditions and numerically tested to show their self-convergence and accuracy (Kim et al., 2003).

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The original GN governing equations were not limited to irrotational flows. The GN model was extended to deep-water waves by Webster and Kim (1991) and Xu et al. (1997) in two and three dimensions, respectively. Demirbilek and Webster (1992) applied the GN theory to shallow water and nonlinear wave propagation problems in two dimensions; this appears to be the first real-world application of the GN theory to prototype environment that have been used in non-idealized bathymetry and wave conditions. Ertekin and Kim (1999) studied the hydroelastic behavior of a three-dimensional mat-type very large floating structure (VLFS) in oblique waves by using the linear Level I GN theory. Xia et al. (2008) studied the two-dimensional nonlinear hydroelasticity of a mat-type VLFS within the scope of linear beam theory for the structure and nonlinear Level I GN theory for the fluid.

The different degrees of GN theory complexity are distinguished by 'levels'. Demirbilek and Webster (1999) applied the GN-2 equations (Level II GN theory) to shallow-water wave problems. Due to the rapid increase in algebraic complexity at higher levels, the GN models up to Level III have been derived but only a limited number of applications exist (Shields and Webster, 1988; Xu et al., 1997). This is even more so in the application of the GN theory to 3-dimensional problems except by the use of the GN-1 equations in a couple of cases (see e.g., Neill and Ertekin, 1997; Ertekin and Sundararaghavan, 2003).

Webster et al. (2011) simplified the structure of the GN equations, thus enabling the utility of the higher-level GN equations (higher than Level III). The Higher-level GN equations have been shown to be more accurate in preliminary numerical calculations (Zhao and Duan, 2012). Different Levels of the classical GN equations have different dispersive properties (see e.g., Webster et al. (2011) for water waves, and Kim and Ertekin (2002) for hydroelastic waves); this is also true for different levels of the IGN equations (see e.g., Kim and Ertekin (2000) and Kim et al. (2003)).

There are two main motivations for this research: to apply the formulation of Webster et al. (2011) for the higher-level GN theory that simplifies the formulation of Demirbilek and Webster (1999) and to solve them accurately and efficiently, and to provide three validation tests for the evaluation of the simplified higher-level GN theory. Note that the GN eqs. of Webster et al. (2011) are essentially the same, for a given level, as previously obtained by others, e.g., Demirbilek and Webster (1992) and Shields and Webster (1988), but they are now manageable computationally because of their much simpler form. Until now, only up to Level 2 GN equations could be used in applications because of their complexity.

In Section 2, the simplified GN equations are introduced. Section 3 provides details of the algorithm, and Section 4 introduces the wave-maker and the damping zone. The test cases that are used in the numerical solution of the higher-level GN equations are presented in Section 5, followed by the conclusions we reach in Section 6.

2. GN theory

The simplified governing equations for the motion of a thin sheet of fluid are provided by the GN theory (Webster et al., 2011). So far, there are basically four sets of equations of the GN theory at each level, these are the ones obtained by Green and Naghdi (1976), by Shields and Webster (1988), by Demirbilek and Webster (1999), and by Webster et al. (2011). They appear different but they are essentially the same as they all follow the main character of the GN theory; that the equations provide solutions that satisfy the exact free-surface and sea-floor boundary conditions, and the postulated conservation laws exactly in the depth-integrated sense. For example, the GN-1 equations of Green and Naghdi (1976) would contain the bottom pressure in the equations but not the GN-1 equations derived by Ertekin (1984) (see e.g., the original GN-1 Eq. (2.9) versus Ertekin's GN-1 Eqs. (2.16), (2.22), and (2.23) in Ertekin (1984)) or by Demirbilek and Webster (1999), although they provide exactly the same solutions. The simplified equations of Webster et al. (2011) makes it possible to solve the higher-level GN equations easily, something that was not possible before as the equations contained many terms with high-order spatial derivatives. The current equations, regardless of the level chosen, have up to third spatial derivatives only.

In this work, only two-dimensional wave problems are considered. x is the horizontal and z is the vertical coordinate and the origin of the coordinate system is located at the still-water level. The two-dimensional GN equations involve the unknowns, β, u_0, u_1, …, where, $z = \beta\ (x,\ t)$is the free surface, u_0, u_1, … are the unknown velocity coefficients and $z = \alpha\ (x)$ represents the sea-floor surface, assumed to be time-independent here. The reduced set of differential equations for Level K GN (GN-K) theory are as follows:

$$\frac{\partial \beta}{\partial t} = \sum_{n=0}^{K} \beta^n \left(w_n - \frac{\partial \beta}{\partial x} u_n \right) \tag{1}$$

$$\frac{\partial}{\partial x}(G_n + gS1_n) + nE_{n-1} - \alpha^n \frac{\partial}{\partial x}(G_0 + gS1_0) = 0 \text{ for } n = 1, 2, 3, \cdots, K \tag{2}$$

where

$$E_n = \sum_{m=0}^{K} \left(\frac{\partial u_m}{\partial t} S2_{mn} + \frac{\partial u_m}{\partial x} Q_{mn} + u_m H_{mn} \right) \tag{3a}$$

$$G_n = \sum_{m=0}^{K} \left(\frac{\partial w_m}{\partial t} S2_{mn} + \frac{\partial w_m}{\partial x} Q_{mn} + w_m H_{mn} \right) \tag{3b}$$

$$Q_{mn} = \sum_{r=0}^{K} u_r S3_{mrn} \tag{3c}$$

$$H_{mn} = \sum_{r=0}^{K} w_r S4_{mrn} \tag{3d}$$

$$S1_n = \int_{\alpha}^{\beta} z^n dz \tag{3e}$$

$$S2_{mn} = \int_{\alpha}^{\beta} z^{m+n} dz \tag{3f}$$

$$S3_{mrn} = \int_{\alpha}^{\beta} z^{m+r+n} dz \tag{3g}$$

$$S4_{mrn} = m \int_{\alpha}^{\beta} z^{m+r+n-1} dz \tag{3h}$$

$$u_K = 0 \tag{3i}$$

$$w_n = -\frac{1}{n} \frac{\partial u_{n-1}}{\partial x} (n = 1, 2, \cdots, K) \tag{3j}$$

$$w_0 = -\sum_{n=1}^{K} \alpha^n \left(w_n - \frac{\partial \alpha}{\partial x} u_n \right) + \frac{\partial \alpha}{\partial x} u_0 \tag{3k}$$

where m, r and n are nonnegative integers and $g = 9.81\ m/s^2$ is the gravitational acceleration, and t is the time.

For the GN-1 equations, the horizontal velocity u and the vertical velocity w are given by (see e.g., Ertekin (1988))

$$u(x, z, t) = u_0(x, t) \tag{4a}$$

$$w(x, z, t) = w_0(x, t) + w_1(x, t) z \tag{4b}$$