Coastal Engineering 80 (2013) 1–14

Contents lists available at SciVerse ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

On the run-up and back-wash processes of single and double solitary waves — An experimental study

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ARTICLE INFO

Article history:
Received 18 October 2012
Received in revised form 29 April 2013
Accepted 1 May 2013
Available online 2 June 2013

Keywords:
Run-up
Solitary wave
Undular bore
Surf parameter
Back-wash
Breaking criterion

ABSTRACT

The run-up and back-wash processes of single and double solitary waves on a slope were studied experimentally. Experiments were conducted in three different wave flumes with four different slopes. For single solitary wave, new experimental data were acquired and, based on the theoretical breaking criterion, a new surf parameter specifically for breaking solitary waves was proposed. An equation to estimate maximum fractional run-up height on a given slope was also proposed. For double solitary waves, new experiments were performed by using two successive solitary waves with equal wave heights; these waves were separated by various durations. The run-up heights of the second wave were found to vary with respect to the separation time. Particle image velocimetry measurements revealed that the intensity of the back-wash flow generated by the first wave strongly affected the run-up height of the second wave. Showing trends similar to that of the second wave run-up heights, both the back-wash breaking process of the first wave and the reflected waves were strongly affected by the wave–wave interaction. Empirical run-up formula for the second solitary wave was also introduced.

1. Introduction

The run-up process of single solitary wave has been extensively studied. For example, Hall and Watts (1953) measured run-up heights for solitary waves with various wave-height-to-water-depth ratios on five different slopes. Synolakis (1987) derived analytical solutions (based on non-linear shallow water equations) describing the evolution of a non-breaking solitary wave on a uniform slope, and validated them with experiments on a 1/19.85 slope. Experiments for breaking solitary waves on the same slope were also conducted in Synolakis' study. More recently, Hsiao et al. (2008) and Chang et al. (2009) investigated solitary wave run-up in a large-scale wave tank on 1/60 and 1/20 slopes, respectively. Other reported experimental data include: Langsholt (1981), Gjevik and Pedersen (1981), Zelt (1991), Briggs et al. (1995), Li and Raichlen (2001, 2002), and Jensen et al. (2003). Various empirical formulas for the maximum run-up height of breaking solitary wave with different range of applicability have been proposed.

Numerically, Svendsen and Grilli (1990) simulated non-linear wave run-ups on steep slopes with a fully non-linear potential-flow-based model, and calculated the back-wash flow during the run-down phase, Zelt (1991) developed a finite element model for solving the Boussinesq equations and calculated run-up heights on various slopes and proposed an artificial bottom-friction factor to monitor run-up heights on different slopes. Solving the fully non-linear free-surface potential flow problem with an integral equation method, Grilli et al. (1997) classified solitary wave breaking types. On the other hand, Li and Raichlen (2002) computed run-up heights for both breaking and non-breaking solitary waves using a shock capturing model. Finally, Kobayashi and Karjadi (1994) and Fuhrman and Madsen (2008) explored the possibility of using a surf parameter to describe the run-up heights of breaking solitary waves.

Compared to the rich literature on single solitary wave run-up, little information on the run-up process of multiple solitary waves is available, which can have important application on estimating the run-up of waves with multiple crests. Undular bores have been observed during tsunami events (e.g., Grue et al., 2008, and Madsen et al., 2008) and can be viewed as a combination of solitary waves with different wave heights and different separation times among them. Peregrine (1966) pointed out that undular bores tend to grow into a succession of solitary waves, while recently El et al. (2012) showed by theory that a sequence of isolated leading solitons forms as an undular bore propagates into decreasing water depth, although the relevant geophysical scale has not been explicitly discussed. Similar discussion has also been made based on numerical and experimental works, i.e. Grilli et al. (2012). Thus, there is a need to perform

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0378-3839/$ – see front matter
http://dx.doi.org/10.1016/j.coastaleng.2013.05.001

controlled experiments so as to gain a better understanding of the run-up process associated with multiple solitary waves.

To make the problem more tractable, as a first step we seek to experimentally study two identical solitary waves separated by various separation times. We remark here that Raichlen (1985) was the first one who performed run-up height measurements on double solitary waves. However, the wave conditions were limited and only two slopes were considered in his experiments. In this study, we examine the run-up and back-wash processes in-depth, with a wide range of wave conditions on different slopes, and extend current knowledge in single solitary wave to the double solitary wave scenario.

Additional experiments for single solitary wave run-up are also performed not only to enrich the data base, but also to serve as references for the double solitary wave experiments. In re-analyzing the run-up height data for single solitary waves, we define a new surf parameter for solitary wave, based on the wave breaking criterion following Carrier and Greenspan's (1958) method. The new surf parameter has a slightly different form from those suggested by Kobayashi and Karjadi (1994) and Fuhrman and Madsen (2008).

In the next section, we shall introduce relevant physical and dimensionless parameters to be used in this study. We will then describe the laboratory set-ups and experimental methods, followed by the presentation of results. The run-up height results for single solitary wave will be discussed first. Special attention shall be paid to the definition of surf parameter characterizing wave breaking and run-up. The approach in analyzing single solitary waves is extended to double solitary waves. Empirical formula for the run-up height ratio of two solitary waves is provided. The back-wash breaking, particle image velocimetry (PIV) measurements, and the reflected waves will then be examined accordingly and their relevance explained.

2. Definitions and parameters

In studying the run-up of single and double solitary waves on a uniform slope the following physical parameters are involved: h is the constant water depth up to the toe of the slope; s is the slope expressed in terms of vertical rise divided by horizontal run; η is the surface elevation from the still water level; H is the solitary wave height in the constant depth region; τ is the separation time between the two solitary wave peaks; and R is the vertical run-up height caused by the waves (R_1 and R_2 represent the run-up heights of the first and the second wave, respectively, when double solitary wave is considered). With the above definitions, relevant normalized parameters include: wave height H/h, separation time τ/T where T is the effective wave period to be defined, run-up R/h or R/H, and relative run-up R_2/R_1.

For a solitary wave traveling in the ξ-direction in a constant depth at speed c, the leading-order solution is well-known:

$$\eta(\xi,t) = H\text{sech}^2[K(\xi - ct)], \quad \text{where} \quad K = \frac{1}{h}\sqrt{\frac{3H}{4h}}, \quad \text{and} \quad c = \sqrt{g(H+h)}, \tag{1}$$

and

$$\lambda = \frac{2\pi}{K}, \quad \text{and} \quad T = \frac{2\pi}{Kc}, \tag{2}$$

can be viewed as the effective wavelength and effective wave period, respectively. While $2H/\lambda$ measures the steepness of the solitary wave front, $l = \lambda/2$ characterizes the effective length of the wave front. Since $L = h/s$ is the distance from the toe of the slope to the still water shoreline, another relevant dimensionless parameter is the horizontal length ratio, l/L, i.e., the inverse of the number of effective lengths of the wave front that the solitary wave must travel on the slope to reach the shoreline. In the case of a solitary wave, l/L can be expressed as

$$\frac{l}{L} = \frac{\lambda}{2L} = \left(\frac{2\pi}{\sqrt{3}}\right)s\left(\frac{H}{h}\right)^{-\frac{1}{2}}. \tag{3}$$

3. Experiments

3.1. Wave flumes

Experiments were performed in three wave flumes with four different beach slopes. The three facilities include two medium-sized wave flumes in the DeFrees Hydraulic Laboratory at Cornell University and the large wave flume in the O. H. Hinsdale Wave Research Laboratory at Oregon State University. In each wave flume a piston-type wave-maker is installed on one end of the flume, and on the other end of the flume is a uniform slope; between the wave-maker and the toe of the slope is a constant depth region. Four different slopes are used in the experiments. Table 1 lists the dimensions, materials, and beach slopes of each wave flume. From herein each experiment is identified by the associated beach slope.

To measure water surface displacement, acoustic wave gages (Banner Engineering S18U) with 0.5 mm manufacturer-specified resolution, were used in the wave flumes at Cornell University; resistance-type wave gages with estimated resolution between 1 mm and 1 cm were employed in the large-scale flume at the O. H. Hinsdale Wave Research Laboratory. Video cameras were placed on top of the slopes to measure wave run-up heights. Grids or fiducial marks on the slope allow conversion of pixels in the videos to the known length.

3.2. Wave generation

By using Goring's (1978) method, single and double solitary waves were generated. Fig. 1 shows a typical wave-maker trajectory for generating a solitary wave (note that the wave-maker's definition of negative displacement is towards the beach slope). Grimshaw's (1971) second-order solitary wave solution is used to check the accuracy of free surface elevation. Overall, solitary wave shapes generated in the experiments are highly accurate; see Fig. 1 for comparisons.

Two successive solitary waves, separated by a specified separation time, were also generated by linearly combining the wave-maker trajectories of two individual solitary waves; the superposition process is illustrated in Fig. 2. The two waves generated in such way are called "double solitary waves" herein.

For separation times $\tau/T > 0.47$, we are successful in generating two identical waves that maintain their forms as they travel down the flume. Fig. 3 shows an example of wave gage records of double solitary waves at different locations. In this figure Grimshaw's solitary wave solution is also plotted. The shortest separation time in the experiments reported herein is $\tau/T = 0.472$. Table 2 lists the wave conditions for the double solitary wave experiments.

3.3. Wave height definition

Since single solitary waves in the experiments maintain constant wave shapes in the constant-depth region, the wave heights in the

Table 1
Flume dimensions and slope materials.

Slope s	Length × width × height	Flume wall material	Slope material
1/2.47	34 m × 0.6 m × 0.9 m	Glass	Glass
1/10	12 m × 0.8 m × 1 m	Glass	Glass
1/12	104 m × 3.7 m × 4.6 m	Concrete	Concrete
1/20	34 m × 0.6 m × 0.9 m	Glass	Glass and styrene (plastic)