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Depth-induced wave breaking in a non-hydrostatic, near-shore wave model

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ABSTRACT

The energy dissipation in the surf-zone due to wave breaking is inherently accounted for in shock-capturing non-hydrostatic wave models, but this requires high vertical resolutions. To allow coarse vertical resolutions a hydrostatic front approximation is suggested. It assumes a hydrostatic pressure distribution at the front of a breaking wave which ensures that the wave front develops a vertical face. Based on the analogy between a hydraulic jump and a turbulent bore, energy dissipation is accounted for by ensuring conservation of mass and momentum. Results are compared with observations of random, uni-directional waves in wave flumes, and to observations of short-crested waves in a wave basin. These demonstrate that the resulting model can resolve the relevant near-shore wave processes in a short-crested wave-field, including wave breaking and wave-driven horizontal circulations.

1. Introduction

Of all the physical processes active in the near shore, wave breaking often dominates the hydrodynamics in the surf zone. It controls wave setup (e.g. Longuet-Higgins and Stewart,1964), drives long-shore currents, rip-currents and undertow (e.g. Longuet-Higgins, 1970; Longuet-Higgins and Stewart, 1964; MacMahan et al., 2006; Svendsen, 1984) and is involved in the generation (or release) of infra-gravity waves (e.g. Battjes et al., 2004; Symonds et al., 1982). It is therefore of paramount importance to accurately include the macro-scale effects of wave breaking into coastal wave models describing near-shore hydrodynamics.

At present, near-shore, non-linear wave models of a phase-resolving nature, are usually based on either a Boussinesq-type formulation or a non-hydrostatic approach. Boussinesq models are well established (e.g. Madsen et al., 1991; Nwogu, 1993; Wei et al., 1995) and have been very successful in applications in near-shore regions. However, to increase accuracy, these models have grown quite involved, thereby complicating the numerical implementation. The non-hydrostatic approach is more recent (e.g. Ma et al., 2012; Stelling and Zijlema, 2003; Yamazaki et al., 2009) and uses an implementation of the basic 3D mass and momentum balance equations for a water body with a free surface. The resulting Euler equations can be supplemented with second-order shear-stress terms when required (resulting in the Navier–Stokes equations). The basic difference with conventional Navier–Stokes models is that the free-surface is described using a single-valued function of the horizontal plane. When compared to more involved methods (e.g. Volume of Fluid or Smoothed Particle Hydrodynamics Dalrymple and Rogers, 2006; Hirt and Nichols, 1981), this allows non-hydrostatic models to efficiently compute free surface flows.

However, neither Boussinesq models nor non-hydrostatic models can be directly applied to details of breaking waves, since in both models essential processes such as overturning, air-entrainment and wave generated turbulence, are absent. But, if only the macro-scale effects of wave breaking are of interest, such as the effect on the statistics of wave heights, details of the breaking process can be ignored. By observing that both spilling and plunging breakers eventually evolve into a quasi-steady bore, where the entire front-face of the wave is turbulent (Peregrine, 1983), a breaking wave becomes analogous to a hydraulic jump (e.g. Lamb, 1932). Consequently, its integral properties (rate of energy dissipation, jump height) are approximately captured by regarding the breaking wave as a discontinuity in the flow variables (free surface, velocities). Proper treatment of such a discontinuity in a non-hydrostatic model (conservation of mass and momentum) can therefore be used to determine the energy dissipation of waves in the surf zone (e.g. Hibberd and Peregrine, 1978).

However, compared to the vertical resolutions which are (1–3 layers) sufficient to describe the wave physics outside the surf zone (e.g. refraction, shoaling, diffraction, non-linear interactions), such a representation of dissipation due to wave breaking requires a disproportional high vertical resolution (~10–20). At low resolutions (<5 layers) the initiation of wave breaking is often delayed when compared to observations, and dissipation in the surf zone is underestimated. Such high resolutions are, at present, not feasibly attainable for extensive

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horizontal domains (say 10×10 wavelengths). Hence, in such cases, an alternative, more efficient approach is required.

To this end we adopt a method which is akin to the approach of Tonelli and Petti (2010) in their Boussinesq model. By enforcing a hydrostatic pressure distribution at the front of a wave, we can locally reduce a non-hydrostatic wave model to the shallow water equations. The wave then rapidly transitions into the characteristic saw-tooth shape and, consistent with the high resolution approach, dissipation is captured by ensuring momentum conservation over the resulting discontinuity. Compared to more involved wave breaking models (e.g. eddy viscosity and surface roller models, Cienfuegos et al., 2010; Kennedy et al., 2000; Schäffer et al., 1993, e.g.), such an approach requires fewer additional parameters (controlling the onset and cessation of wave dissipation) and is easily extendible to two horizontal dimensions.

In the present work we will demonstrate that: (a) with a sufficiently high vertical resolution non-hydrostatic models can properly determine the dissipation of breaking waves *without* additional model assumptions; and (b) that similar results –at significantly reduced computational cost – can be obtained with a more practical low vertical resolution, *if* locally the non-hydrostatic model is reduced to a hydrostatic model. Furthermore, by comparison to experimental data (Dingemans et al., 1986), we will show that, in contrast to a high resolution model, the latter can be feasibly applied to situations with short-crested waves over two-dimensional topography, including the wave generated mean currents.

This paper is organized as follows: Section 2 introduces the basic equations that govern the non-hydrostatic model SWASH[1] (Zijlema et al., 2011) and briefly addresses the relevant details of its numerical implementation. In Section 3 the hydrostatic front approximation is introduced and in Section 4 the parameter that controls the onset of wave breaking is estimated from experimental data. Section 5 compares significant wave heights and mean periods obtained from computations using high and low vertical resolutions (using the hydrostatic front approximation) with measured data from flume experiments. Subsequently, in Section 6 the approximate model is compared to the experiment by Dingemans et al. (1986). Finally, we discuss our results and summarize our main findings in Sections 7 and 8.

2. Non-hydrostatic modelling

2.1. Governing equations

The non-hydrostatic model SWASH (Zijlema et al., 2011), is an implementation of the basic 3D mass and momentum balance of a free surface, incompressible fluid with constant density. For reasons of exposition, we present these equations for the 2D vertical plane (the extension to full 3D is straightforward). In terms of Cartesian coordinates *x*,*z* (defined in and normal to the still water level, with *z* positive upwards, respectively) and time *t*, these equations are

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial wu}{\partial z}=-\frac{1}{\rho}\frac{\partial(p_n+p_{nh})}{\partial x}+\frac{\partial\tau_{xz}}{\partial z}+\frac{\partial\tau_{xx}}{\partial x} \quad (1)$$

$$\frac{\partial w}{\partial t}+\frac{\partial uw}{\partial x}+\frac{\partial ww}{\partial z}=-\frac{1}{\rho}\frac{\partial p_{nh}}{\partial z}+\frac{\partial\tau_{zz}}{\partial z}+\frac{\partial\tau_{zx}}{\partial x} \quad (2)$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \quad (3)$$

in which $u(x,z,t)$ and $w(x,z,t)$ are the horizontal and vertical velocities, respectively; ρ is density; p_h and p_{nh} are the hydrostatic and non-hydrostatic pressures, respectively, and $\tau_{xx}, \tau_{xz}, \tau_{zz}, \tau_{zx}$ are the turbulent stresses. The water column is vertically restricted by the moving free-surface $\zeta(x,t)$ and stationary bottom $d(x)$ (measured positive downwards), defined relative to the still water level z_0. Furthermore, the hydrostatic pressure is explicitly expressed in terms of the free-surface level as $p_h=\rho g(\zeta-z)$ so that $\partial_z p_h=-g\rho$ (where ∂_z is short for $\partial/\partial z$) and $\partial_x p_h=\rho g\partial_x\zeta$ (with $g=9.81m/s^2$ the gravitational acceleration). Finally the time evolution of the surface elevation is obtained by considering the mass (or volume) balance for the entire water column

$$\frac{\partial\zeta}{\partial t}+\frac{\partial}{\partial x}\int_{-d}^{\zeta}udz=0. \quad (4)$$

For waves propagating over intermediate distances (say *O(10)* characteristic wavelengths), in the absence of strongly sheared currents, the influence of (turbulent-) stresses on the wave motion proper is relatively small (e.g. Mei et al., 2005, Section 9) and can – to a good approximation – be neglected. However, to allow the influence of bottom friction to extend over the vertical (important for low frequency motions), some vertical mixing is introduced by means of stress terms based upon a turbulent viscosity approximation (e.g. $\tau_{xz}=\nu_v\partial_z u$, with ν_v the vertical mixing coefficient). This also introduces additional vertical coupling, thereby increasing numerical stability. Depending on how well the vertical is resolved the eddy viscosity is estimated with either a constant value, or obtained from a more elaborate $k-\varepsilon$ closure model (see Appendix A). Finally, to allow for lateral mixing in horizontal circulations due to sub-grid eddies, lateral stresses are added (see Appendix A).

Eqs. (1)–(4) are solved for kinematic and dynamic boundary conditions at the surface and at the bottom. The kinematic boundary conditions are that no particle shall leave the surface and no particle shall penetrate the (fixed) bottom, giving

$$w=\frac{\partial\zeta}{\partial t}+u\frac{\partial\zeta}{\partial x} \qquad \text{at} \qquad z=\zeta(x,t) \quad (5)$$

$$w=-u\frac{\partial d}{\partial x} \qquad \text{at} \qquad z=-d(x). \quad (6)$$

The dynamic boundary condition at the surface is a constant pressure (zero atmospheric pressure, $p_h=p_{nh}=0$) and no surface stresses. Since bottom friction is important for low frequency motions and during wave run-up, a stress term at the bottom boundary τ_b is added based on a quadratic friction law

$$\tau_b=c_f\frac{U|U|}{h} \quad (7)$$

where U is the depth averaged current, $h=d+\zeta$ the water depth and c_f is a (dimensionless) friction coefficient. A correct estimation of c_f is important in case of steady horizontal circulations, where the bottom friction balances the forcing by the radiation stress. Here we obtain the friction coefficient from the Manning–Strickler formulation as $c_f=0.015(k/h)^{1/3}$, where k is an (apparent) roughness value. In the presence of waves this apparent roughness value is not necessarily related to the bottom roughness. In particular in the surf zone, observations have shown that the apparent roughness (or friction coefficient) can be substantially higher (e.g. Feddersen et al., 2003), because the near bed vertical flux of horizontal momentum is influenced by the breaking induced turbulence. For simplicity, near-shore circulation models often do not separately account for this. Instead, they assume a constant c_f, which is obtained by comparison to observations (e.g. Chen et al., 1999; Ozkan-Haller and Kirby, 1999; Tonelli and Petti, 2012, among many others). In the present study – for flume-type situations – k is estimated based upon the surface properties. However, in case of a mean horizontal circulation, we likewise obtain the apparent roughness after calibration.

We assume that the domain is bound horizontally by a rectangular boundary that consists of four vertical planes. Waves are generated by prescribing the horizontal particle velocities normal to the boundary over the vertical. These velocities are either obtained from time-series for each point on the boundary, or by imposing a wave spectrum

[1] Simulation WAves till SHore, freely available at http://swash.sourceforge.net/.