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Enhancing energy harvesting by a linear stochastic oscillator

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ABSTRACT

A mathematical model of a linear electromechanical oscillator with a randomly fluctuating damping parameter as an energy harvester is considered. To demonstrate the idea and present an explicit analytical solution the variations of the damping are taken in a form of a telegraphic process. It is shown that substantial enhancement of the harvested energy can be reached if the proposed process has specially selected characteristics. A close relationship with the mean square stability of the oscillator is established. An analytical result for stationary electrical net mean power is presented.

1. Introduction

Energy harvesting is a conversion of ambient energy presented in the environment into electrical energy that can be used or stored. The energy harvesters can be used as a replacement for batteries in low-power wireless electronic devices (e.g. sensor networks used in structural health monitoring). Among the power harvesting methods, the conversion of mechanical energy from vibrations into electrical energy via the piezoelectric effect is particularly effective and has received a great attention over the last 10 years (see e.g. books [1,2] and reviews [3–5]. Vibration-based energy harvesters are typically implemented as linear mechanical resonators based on the following well-known phenomenon which is usually described at undergraduate texts in differential equations.

Consider the ubiquitous harmonic oscillator

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = \eta(t), \tag{1}$$

where $\gamma > 0$ is a damping parameter, ω is the natural frequency, the excitation $\eta(t)$ is harmonic, $\eta(t) = a\sin(\Omega t)$. Then a solutions of Eq. (1) grows dramatically if $\omega \approx \Omega$ and γ is small (the resonance phenomenon). A crucial limitation of the mechanical resonators is that the best performance of the devices is limited to a very narrow bandwidth around the natural frequency. If the excitation frequency deviates slightly from the resonance condition, the power output is drastically reduced, rendering linear resonant harvesters unsuitable for most practical applications. In a lot of cases the excitation exhibits a random rather than deterministic behavior with a broad-band frequency spectrum.

When a broad-band external excitation is modeled by a zero-mean Gaussian white noise with intensity D, the power gained by the linear system is proportional to the noise intensity $P_w = D$. The latter is a very interesting result, since the harvested energy of a linear system is independent of the system's parameters. This fact has been extended to the case of nonlinear systems [6] and it has been shown that the mass of the system and the noise intensity are the two factors influencing the harvesting capabilities of any dynamical system, except the bistable systems, for instance [7–10]. It should be stressed that the above result is the upper boundary for the considered class of systems which may or may not be reached, whereas in the real life environment, where the excitation is rather broad band with finite power, any system has to be adjusted for its best performance. For instance if one considers a linear system (1) under the Ornstein–Uhlenbeck process then power of (1) can be obtained exactly analytically using the method of moments

$$P_\mu = \frac{D(1+\gamma\mu)}{(1+\gamma\mu+\omega^2\mu^2)} = P_w \frac{1}{1+\frac{\omega^2\mu^2}{1+\gamma\mu}}$$

where D, μ are parameters of the Ornstein–Uhlenbeck process. The above formula clearly shows that the harvested power depends on both the damping coefficient and the natural frequency of the system.

In this paper we propose a promising approach for the vibration-based energy harvesting staying within the linear mechanical system framework. The vibratory system has random time-varying variations of the damping parameter with specially selected characteristics that allow significantly enhance the energy

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harvesting. It can be achieved by using a magnetorheological (MR) damper [11]. The major idea behind the MR damper is the MR fluid which can change its viscosity under the influence of a magnetic field. It has been reported that MR dampers require relatively low amount of energy, outperform passive devices and SMAs, as well as have a fast response [12–15].

2. Mathematical model of the energy harvester

The vibration-based energy harvester comprises two parts. The key part is a mechanical oscillator but the second one is a capacitor that stores the electrical energy and is coupled with the oscillator by a transducer mechanism based on the piezoelectricity. A simplified representation of the piezoelectric vibration – based energy harvester is shown in Fig. 1 where the left part is the capacitive energy harvesting circuit (capacitor) but the right one is the base accelerated mechanical oscillator. Using Newton's second law and Ohm's law one can obtain for the mechanical states and electric voltage the following coupled equations [16]:

$$m\frac{d^2\bar{x}(\tau)}{d\tau^2} + b(\tau)\frac{d\bar{x}(\tau)}{d\tau} + k\bar{x}(\tau) + \theta\bar{V}(\tau) = -m\frac{d^2\bar{Z}(\tau)}{d\tau^2}, \tag{2}$$

$$RC_p\frac{d\bar{V}(\tau)}{d\tau} + \bar{V}(\tau) = R\theta\frac{d\bar{x}(\tau)}{d\tau}, \tag{3}$$

where $\bar{x}$ is a deflection of mass m, $b(\tau)$ is a time-varying damping coefficient, k is a stiffness coefficient, θ is a linear electromechanical coupling, C_p is a capacitance of the piezoelectric element, $\bar{V}$ is an electric voltage measured across the equivalent resistive load R, and $\frac{d^2\bar{Z}(\tau)}{d\tau^2}$ is a base acceleration. We suppose that the damping coefficient has the form

$$b(\tau) = a[1 + \sigma\zeta(\omega_1\tau)],$$

where $\zeta(\omega_1\tau)$ is some narrow band random variations of the constant damping coefficient a with a frequency ω_1 and noise intensity $0 \le \sigma < 1$. Introduce the substitutions

$$\omega = \sqrt{\frac{k}{m}}, \quad t = \omega\tau, \quad \gamma = \frac{a}{\sqrt{mk}}, \quad k_1 = \frac{\theta^2}{kC_p}, \quad k_2 = \frac{1}{RC_p\omega},$$
$$x_1 = \frac{\bar{x}}{l_c}, \quad Z = \frac{\bar{Z}}{l_c}, \quad x_3 = \frac{\bar{V}C_p}{\theta l_c}$$
$$x_2 = \frac{dx_1}{dt}, \quad \eta = -\frac{d^2Z}{dt^2}, \quad \xi(t) = \zeta\left(\frac{\omega_1 t}{\omega}\right), \tag{4}$$

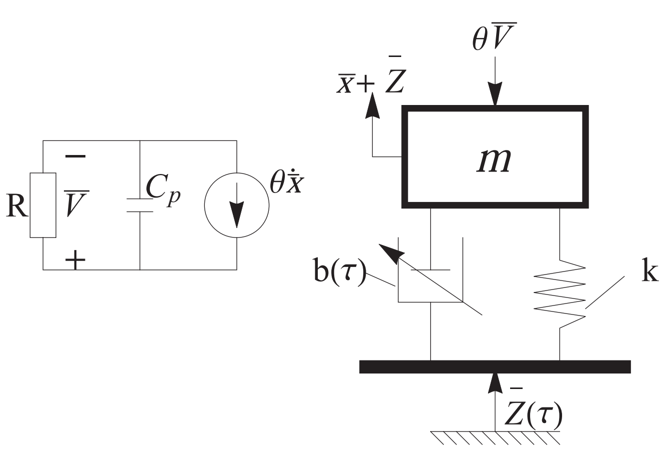

Fig. 1. A simplified representation of the vibration-based piezoelectric energy harvester.

where l_c is a length scale. Then Eqs. (2) and (3) are reduced to the following system in the non-dimensional form:

$$\frac{dx_1}{dt} = x_2,$$
$$\frac{dx_2}{dt} = -x_1 - \gamma[1 + \sigma\xi(t)]x_2 - k_1x_3 + \eta(t),$$
$$\frac{dx_3}{dt} = x_2 - k_2x_3, \tag{5}$$

We suppose that the process $\eta(t)$ is a Gaussian white noise with intensity D. This process is considered as a good model of a broadband ambient excitation (see e.g. [17,16,18,19]). The random process $\xi(t)$ is a zero-mean telegraphic process with the state $\{-1, 1\}$ and the transition rate $\alpha/2$. This process is an ergodic Markov process and it is often used in applications (see e.g. books [20,21]). The spectral characteristics of system (5) can be obtained exactly using the measuring filters approach [22]. It should be stressed that such a choice of the process was dictated by the ability of deriving an exact analytical solution to set of Eqs. (5) rather than anything else, thereby illustrating clearly the influence of parameters on the energy harvesting process. It is possible to consider any other narrow band exponentially correlated process excitation process, but an explicit analytical solution may not be available resulting in a lack of understanding the phenomenon behind the proposed idea. The processes $\eta(t)$ and $\xi(t)$ are supposed independent. System (4) can be presented as a linear system of Itô stochastic differential equations

$$dx_1 = x_2\,dt,$$
$$dx_2 = -x_1\,dt - \gamma[1 + \sigma\xi(t)]x_2\,dt - k_1x_3\,dt + D\,dw(t),$$
$$dx_3 = x_2\,dt - k_2x_3\,dt, \tag{6}$$

where $w(t)$ is a standard Wiener process.

3. Analytical development and results

First consider system (4) with $\eta \equiv 0$. The trivial solution of this system is called mean square asymptotically stable if all second moments of any solution of the system tend to zero when $t \to \infty$. Let

$$\alpha_1 = \alpha + \gamma$$
$$a_0 = \alpha_1(1 + \alpha(1 + \alpha_1 + k_1) + [\alpha_1(2\alpha + \gamma) + k_1]k_2$$
$$+ \alpha_1k_2^2 \times [\gamma^2k_2 + k_1k_2 + \gamma(1 + k_1 + k_2^2)][8k_2 + 4\alpha(1 + k_1)$$
$$+ \alpha(\alpha + 2\gamma)(\alpha + 2k_2)],$$
$$a_1 = b_0 + b_1k_2 + b_2k_2^2 + b_3k_2^3 + b_4k_2^4,$$
$$b_0 = 2\alpha(1 + k_1)[\alpha^4 + 2\alpha^3\gamma + 3\alpha\gamma(1 + k_1) + 2(1 + k_1)^2$$
$$+ \alpha^2(3 + \gamma^2 + 3k_1)],$$
$$b_1 = 3\alpha^5\gamma + 8(1 + k_1)^2 + \alpha^4(8 + 7\gamma^2 + 4k_1)$$
$$+ \alpha^3\gamma \times (24 + 4\gamma^2 + 17k_1) + 4\alpha\gamma(5 + 8k_1 + 3k_1^2)$$
$$+ 2\alpha^2(10 + 13k_1 + 3k_1^2 + 8\gamma^2 + 7\gamma^2k_1),$$
$$b_2 = 2\alpha^5 + 15\alpha^4\gamma + 16\gamma(1 + k_1) + 4\alpha^2\gamma(13 + 3\gamma^2 + 5k_1)$$
$$+ 8\alpha[3 + 2k_1 + 2\gamma^2(2 + k_1)] + \alpha^3[25\gamma^2 + 4(4 + k_1)],$$
$$b_3 = 2[4\alpha^4 + 13\alpha^3\gamma + 8(1 + \gamma^2) + 4\alpha\gamma(6 + \gamma^2 + k_1)$$
$$+ \alpha^2 \times (13\gamma^2 + 12 + 3k_1)],$$
$$b_4 = 2[5\alpha^3 + 8\gamma + 9\alpha^2\gamma + 2\alpha(5 + 2\gamma^2 + k_1)],$$
$$a_2 = 2k_2(\alpha + 2k_2)[\alpha^3 + 2k_2 + \alpha^2(\gamma + k_2) + \alpha(2 + k_1 + \gamma k_2)]. \tag{7}$$

It should be emphasized that the damping coefficient γ is always positive because of our original assumption for $0 < \sigma < 1$.