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The constrained optimisation of small linear arrays of heaving point absorbers. Part I: The influence of spacing

Justin P.L. McGuinness [a,b,*], Gareth Thomas [a]

[a] Department of Applied Mathematics, University College Cork, Ireland
[b] Department of Mathematics, Cork Institute of Technology, Ireland

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ABSTRACT

This paper describes the optimisation of small arrays of Wave Energy Converters (WECs) of point absorber type. The WECs are spherical in shape and operate in heave alone and a linear array of five devices is considered. Previous work is extended by considering the constrained performance of the array members, where an uaniper limit on WEC displacements is enforced. Two opimisations are performed. In each case, the objective function is defined as the mean of the averaged interaction factor ovehe non-dimensional length of the array. The first considers the array layout fixed at a geometry previously identified as optimal in an unconstrained regime and optimises the displacements of the WECs subject to constraints. The second allows both the WEC positions and displacements to vary as optimisation variables. It is shown that the optimal layout of the constrained arrays is different from the unconstrained case. Applying constrained motions results in optimal layouts that are more separated, with less grouping of WECs and this will have practical considerations. The effect of the constraints varies depending on the incident wave angle. In some cases, performance is reduced drastically and stability of performance is improved, while in other cases there is a degradation of performance. Thus, a trade-off between performance and stability of performance is seen when displacement constraints are applied.

1. Introduction

The fundamental modelling of arrays of wave power devices of point absorber type was presented independently in [1,2]. The point absorber approximation assumes that the ratio of device size to incident wavelength is small enough for the scattered wave field of the device to be neglected. This allows a simplification of the calculations, particularly those relating to WEC arrays. Subsequent papers have applied this theory to assess arrays of differing configurations or array properties, e.g. [3–8].

In [1–3], the devices were assumed to be equally spaced and the concept of positive and negative interference within the array was established. The concept of unequal spacing in a linear array was first considered in [4] and it was shown that unequally spaced arrays performed better in some cases in comparison to equally spaced arrays. However, only a very specific case of unequal spacing was considered. The accuracy of the point absorber approximation is discussed in [5], where it is shown that the approximation gives agreement with the exact multiple scattering method for a non-dimensional device radius of $ka < 0.8$. The extension to arbitrary array arrangements, without any stipulated geometry

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[*] Corresponding author at: Department of Applied Mathematics, University College Cork, Ireland.
E-mail addresses: j.mcguinness@umail.ucc.ie (J.P.L. McGuinness), g.thomas@ucc.ie (G. Thomas).

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or symmetry is considered in [6–8]. In [6,7], the point absorber approximation is applied and the interaction factor is numerically maximised with respect to WEC positions for both constrained and unconstrained WEC motions. A full interaction regime is implemented in [8] and the array performance is maximised using a genetic algorithm for both regular and irregular waves.

A major common finding of the previous array optimisation studies (e.g. [3,6–8]) is that the optimal array arrangements were often found to be only slightly different to those corresponding to very poorly performing arrays. In many cases, either the best and worst array layouts were surprisingly close or the optimal array had a sharp peak in performance surrounded by large troughs. This means that a small change in the non-dimensional parameters of such arrays, either by a physical misalignment or a change in sea conditions (incident wavelength or wave angle), can have a potentially disastrous impact on array performance.

This issue was addressed in [9,10], which considered the optimisation of linear and circular arrays of five to seven WECs, where the mean of the interaction factor was maximised, rather than the interaction factor itself. In these works, the mean was taken over a non-dimensional length/radius measure, which resulted in arrays that were stable to changes in non-dimensional separation parameters. However, in some cases, these optimal arrays were still quite sensitive to changes in incident wave angle. One important issue is whether high performance or stability (reliability) of performance is more desirable. Ideally, both would be achieved by an optimal array, however this may not be possible, particularly with the application of WEC motion constraints.

A main concern when considering array performance is the motions of the individual devices associated with optimal performance. A hydrodynamically optimised array is typically accompanied by large amplitude device motions; this is highlighted in [10]. The large motion of WECs creates engineering difficulties with the control, maintenance and power take-off of the devices. In addition, linear wave theory assumes that all device motions are at most of the same order of the wave amplitude, and violation of this requirement invalidates the underlying assumptions; this is considered in [3,6,9,10], where the optimal arrays were predicted to exhibit large device motions. Device motion constraints were investigated in [3,6], where it was found that, in some cases, these constraints severely limited array performance.

The main aim of this paper is the constrained optimisation of WEC arrays such that the resulting optimal array is stable to changes in array parameters. Having an array that performs well in certain conditions but that is also highly sensitive to changes in wavelength or wave angle is not ideal. Wave conditions in the open ocean can change slightly and ideally a WEC array should maintain optimal or at least near-optimal performance in the case of any such changes. Previous research of the nature is extended by considering constrained performance of the WECs, where the WEC motions are limited to two or three times the incident wave amplitude, as in [3,6]. The effect of these constraints are firstly analysed with respect to layouts previously optimised without constraints. The layouts are then re-optimised within the constrained regime and the resulting layouts compared.

The work presented herein is conducted within the regime of validity of the point absorber approximation ($ka < 0.8$) as identified in [5], and the non-dimensional radius of the WECs is fixed at $ka = 0.4$. An external model is required in this methodology to determine the device motions and for the chosen device geometry, which is spherical in this case, the motions can be determined using the approach of [11], for a fixed non-dimensional radius of the WECs.

This research is motivated by the possibility that unequally spaced linear arrays may perform better that their equally spaced analogs. The work presented in [9,10] was similarly motivated, where linear and circular array geometries were enforced and the mean array performance was maximised with respect to the non-dimensional WEC separations. The mean performance was defined over a range of non-dimensional array length or radius.

Section 2 outlines the mathematical theory behind this research, including the definition of the averaged interaction factor for constrained motions and the optimisation method. The results of the optimisation are presented in Section 3. The constrained performance of previously identified unconstrained optimal array layouts is assessed in Section 3.1. In Section 3.2, the array layout is not prescribed and an optimisation over both the WEC motions and positions is performed with respect to the mean of the averaged interaction factor. Finally, a discussion of the results is given in Section 4 along with some conclusions thereof.

2. Mathematical formulation

2.1. Power absorption theory

Consider a linear array of physical length L with N semi-submerged spheres, considered to be point absorbers and which operate in heave alone. It is assumed that linear wave theory is applicable and that regular long-crested waves of amplitude A, frequency ω, wavenumber k and angle β are incident on the array in water of infinite depth, where β is measured in an anticlockwise direction from the positive x-axis. A detailed description of the background theory is available in [9,10], where array power absorption theory and the point absorber approximation are outlined. In this work, constrained motions are considered and the full power absorption equation is employed without the assumption of optimal motions. As shown in [1], the mean power absorbed by the array is

$$P_{abs} = \frac{1}{8}\mathbf{X}^{\dagger}\mathbb{B}^{-1}\mathbf{X} - \frac{1}{2}\left(\mathbf{U} - \frac{1}{2}\mathbb{B}^{-1}\mathbf{X}\right)^{\dagger}\mathbb{B}\left(\mathbf{U} - \frac{1}{2}\mathbb{B}^{-1}\mathbf{X}\right), \tag{1}$$

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