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Transient stochastic response of quasi non-integerable Hamiltonian system

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ABSTRACT

The approximate transient response of multi-degree-of-freedom (MDOF) quasi non-integrable Hamiltonian system under Gaussian white noise excitation is investigated. First, the averaged Itô equation for Hamiltonian and the associated Fokker–Planck–Kolmogorov (FPK) equation governing the transient probability density of Hamiltonian of the system are derived by applying the stochastic averaging method for quasi non-integrable Hamiltonian systems. Then, the approximate solution of the transient probability density of Hamiltonian is obtained by applying the Galerkin method to solve the FPK equation. The approximate transient solution is expressed as a series in terms of properly selected basis functions with time-dependent coefficients. The transient probability densities of displacements and velocities can be derived from that of Hamiltonian. One example is given to illustrate the application of the proposed procedure. It is shown that the results for the example obtained by using the proposed procedure agree well with those from Monte Carlo simulation of the original system.

1. Introduction

The randomness in structural dynamics is often encountered in various engineering fields. The sources of randomness may be environmental loads such as ground motion, atmospheric turbulence, sea waves, etc., as well as structural properties such as materials properties, geometry parameters and so on. Moreover, real structures are generally nonlinear and of MDOF. In recent years, the response of nonlinear systems to stochastic excitation has received much attention. If a nonlinear system is subjected to Gaussian white noise excitation, the probability density function (PDF) of the response is governed by the Fokker–Planck–Kolmogorov (FPK) equation. It is difficult to obtain the transient solution of the FPK equation for MDOF nonlinear stochastic system. The analytical transient solutions of the FPK equation are available only for linear systems and some special nonlinear systems [1–3]. Due to the difficulty in solving FPK equation, the Monte Carlo simulation method is commonly used to analysis the transient responses of nonlinear stochastic systems. Some other numerical methods have also been proposed to predict the transient responses of nonlinear systems under stochastic excitations, e.g., the cell mapping method [4], path integration method [5,6], and finite element method [7]. Recently, Pichler used the finite element method to compute the evolution of the joint probability density function over time and to determine first excursion probabilities [8]. Yue et.al. studied the transient and steady-state responses in a self-sustained oscillator subject to harmonic and bounded noise excitations using generalized cell mapping method [9]. Iwan and Spanos obtained the nonstationary amplitude response statistics of a lightly damped and weakly nonlinear oscillator subject to Gaussian white noise by using averaging method and perturbation techniques [10]. The Galerkin method, as a variation method, has been adopted to study both transient responses and first passage problems[11,12] of nonlinear systems. Atkinson adopted this method to study the stationary response of several second-order nonlinear systems [13], and the non-stationary case was studied by Wen [14]. Recently, the non-stationary response of nonlinear oscillators subject to additive Gaussian white noise was studied by Spanos and the transient probability density of the system has been obtained as the sum of a set of selected basis functions with time-dependent coefficients through the general Galerkin method [15]. Using this method, Jin et. al. studied the non-stationary probability density of single-degree- of-freedom (SDOF) strongly nonlinear system subject to modulated white noise excitation [16], the non-stationary probability densities of MDOF nonlinear systems under Gaussian white noise excitations [17,18], and the first-passage failure of MDOF nonlinear oscillator [19]. Qi and Xu et. al. studied the non-stationary probability densities of a class of nonlinear systems excited by external colored noise [20]. However, the results obtained by using the Galerkin method are limited to

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SDOF systems or MDOF weakly coupled systems under stochastic excitations. Recently, the transient responses of quasi integrable Hamiltonian systems [21] and quasi partially integrable Hamiltonian systems [22] have been studied by the present authors.

The stochastic averaging method [23,24] may be used to reduce the dimension of stochastic systems. It has been applied by many authors to study the stationary responses, the stabilities with probability one, the first-passage problem for MDOF nonlinear stochastic systems. Recently, the stochastic averaging method has been extended by Zhu and Yang [25] to MDOF quasi non-integrable Hamiltonian systems. Moreover, the stationary response [25], stochastic stability [26], first passage failure [27] and Hopf bifurcation [28] of MDOF nonlinear stochastic systems has also been studied by using the stochastic averaging method for quasi non-integrable Hamiltonian systems.

In the present paper, the approximate transient response of quasi non-integable Hamiltonian system under Gaussian white noise excitation is investigated. First, the averaged Itô equation of Hamiltonian and the associated FPK equation governing the transient probability density of Hamiltonian are derived by applying the stochastic averaging method for quasi non-integrable Hamiltonian systems. Then, the approximate solution of the transient probability density of the Hamiltonian is obtained by applying the Galerkin method to solve the FPK equation. The approximate solution is expressed as a series in terms of properly selected base functions with time-dependent coefficients. The transient probability densities of displacements and velocities can be derived from that of Hamiltonian. Finally, an example is given to illustrate the application of the proposed procedure. It is shown that the results obtained for the example by using the proposed procedure agree well with those from Monte Carlo simulation of the original system.

2. The stochastic averaging method for quasi non-integrable Hamiltonian systems

Consider an n degree-of-freedom (DOF) stochastically excited and dissipated Hamiltonian system governed by the following n pairs of equations:

$$\dot{Q}_i = \frac{\partial H'}{\partial P_i} \tag{1a}$$

$$\dot{P}_i = -\frac{\partial H'}{\partial Q_i} - \varepsilon c_{ij}\frac{\partial H'}{\partial P_i} + \varepsilon^{1/2} f_{ik} W_k(t) \tag{1b}$$

$i, j = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, m.$

where Q_i and P_i are generalized displacements and generalized momenta, respectively; $H' = H'(\mathbf{Q}, \mathbf{P})$ is a Hamiltonian with continuous second-order derivatives; $c_{ij} = c_{ij}(\mathbf{Q}, \mathbf{P})$ are differentiable functions representing coefficients of quasi linear dampings; $f_{ik} = f_{ik}(\mathbf{Q}, \mathbf{P})$ are twice-differentiable functions denoting amplitudes of random excitations; ε is a small positive parameter; $W_k(t)$ are Gaussian white noises in the sense of Stratonovich with correlation functions

$$E[W_k(t)W_l(t+\tau)] = 2D_{kl}\delta(\tau) \tag{2}$$

The system governed by Eqs. (1a) and (1b) is termed a quasi-Hamiltonian one and it is generally nonlinear. The first summation terms on the right-hand side of Eq. (1b) may represent a set of linear and (or) nonlinear damping mechanisms, while the second summation terms may include external and (or) parametric excitations of Gaussian white noise.

Eqs. (1a) and (1b) are equivalent to the following set of Itô stochastic differential equations:

$$dQ_i = \frac{\partial H'}{\partial P_i} dt \tag{3a}$$

$$dP_i = \left(-\frac{\partial H'}{\partial Q_i} - \varepsilon c_{ij}\frac{\partial H'}{\partial P_i} + \varepsilon D_{kl} f_{jl}\frac{\partial f_{ik}}{\partial P_j}\right)dt + \varepsilon^{1/2} f_{ik}\, dB_k(t) \tag{3b}$$

$i, j = 1, 2, \ldots, n;$ k$= 1, 2, \ldots, m.$

where $B_k(t)$ are the Wiener processes. The double summation terms on the right-hand side of Eq. (3b) are known as the Wong–Zakai correct terms [29]. These terms can usually be split into two parts: one has the effect of modifying the conservative forces and another modifying the damping forces. The first part can be combined with $-\partial H'/\partial Q_i$ to form overall effective conservative force $-\partial H/\partial Q_i$ with modified Hamiltonian $H = H(\mathbf{Q}, \mathbf{P})$ and with $\partial H/\partial P_i = \partial H'/\partial P_i$. The second part may be combined with $-\varepsilon c_{ij}\partial H'/\partial P_j$ to constitute effective damping forces $-\varepsilon m_{ij}\partial H/\partial P_j$ with $m_{ij} = m_{ij}(\mathbf{Q}, \mathbf{P})$. With these accomplished, Eqs. (3a) and (3b) can be rewritten as follows:

$$dQ_i = \frac{\partial H}{\partial P_i} dt \tag{4a}$$

$$dP_i = -\left(\frac{\partial H}{\partial Q_i} + \varepsilon m_{ij}\frac{\partial H}{\partial P_i}\right)dt + \varepsilon^{1/2} f_{ik}\, dB_k(t) \tag{4b}$$

$i, j = 1, 2, \ldots, n;$ k$= 1, 2, \ldots, m.$

Without dampings and Gaussian white noise excitations, Eqs. (4a) and (4b) become

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \tag{5a}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \tag{5b}$$

$i = 1, 2, \ldots, n$

which represent an n-DOF Hamiltonian system with modified Hamiltonian H. It is well known that a Hamiltonian system may be integrable or nonintegrable. An n-DOF Hamiltonian system is said to be integrable or completely integrable if there exist n independent integrals of the motion which are in involution. This last term means that the Poisson bracket of any two these integrals vanishes. For a nonintegrable or completely non-integrable Hamiltonian system, this is only one independent integral of the motion, i.e., the Hamiltonian H. Completely integrable Hamiltonian systems are exceptional. Siegel and Morse [30] have shown that, in certain classes of Hamiltonian systems, the non-integrable ones form a dense set. In view of this fact, in the present paper, the Hamiltonian system governed by Eqs. (5a) and (5b) is assumed to be non-integrable.

Since H is a function of Q_i and P_i, an Itô stochastic differential equation for H can be derived from Eqs. (4a) and (4b) by using Itô differential rule as follow:

$$dH = \varepsilon\left(-m_{ij}\frac{\partial H}{\partial P_i}\frac{\partial H}{\partial P_j} + D_{kl} f_{ik} f_{jl}\frac{\partial^2 H}{\partial P_i \partial P_j}\right)dt + \varepsilon^{1/2}\frac{\partial H}{\partial P_i} f_{ik}\, dB_k(t) \tag{6}$$