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An efficient space–time based simulation approach of wind velocity field with embedded conditional interpolation for unevenly spaced locations

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ABSTRACT

The classical spectral representation method (SRM) has been extensively used in the simulation of multivariate stationary Gaussian random processes. Due to the application of fast Fourier transform (FFT), the simulation is usually efficient. However, for processes with a large number of simulation points, it becomes necessary to enhance the simulation efficiency. One example is the wind velocity field along a large-span bridge, where hundreds of wind velocity fluctuations are required. In the case of bridges built over a homogeneous terrain such as coastal area or flat plain, the wind velocity field can be modeled as a multivariate homogeneous random process i.e., the auto power spectral densities (PSDs) at evenly-spaced simulation points are same and the cross PSD is a function of separation distance between two simulation points. Furthermore, in some applications, additional simulation points need to be included to a set of uniformly distributed points in order to make the wind velocity field consistent to the structural dynamic analysis requirement.

In this paper, a hybrid approach of space–time random-field based SRM and proper orthogonal decomposition (POD)-based interpolation is developed for simulating the above wind velocity process. In this approach, the random-field based SRM is used to simulate the multivariate homogeneous random process composed of a set of uniformly distributed simulation locations while POD-based interpolation is used to conditionally generate the wind velocities at a few unevenly distributed points using the previously simulated wind velocities. The idea of the former is based on transforming the simulation of the homogeneous random process into that of the corresponding space–time random field where the phase angle is assumed to be zero and the coherence function must be an even function in terms of separation distance. Through this procedure, customary requirement for spectral matrix decomposition is eliminated and application of two dimensional FFT can improve the simulation efficiency dramatically. The shortcomings of this method include a slight approximation regarding the simulated sample and the non-ergodicity for the correlation function. The numerical example of a homogeneous wind velocity field along a bridge deck shows that the proposed random field-based method is very efficient in terms of accuracy and efficiency when the number of simulation locations is large and the POD-based interpolation also has good performance.

1. Introduction

The frequency domain based analysis approach is generally preferred in engineering practice, but Monte Carlo simulation is still widely used when nonlinearity, system stochasticity, and other related problems exist. For example, Monte Carlo simulation approach is more convenient for considering the structural and aerodynamic nonlinearities in the wind-induced dynamic analysis of long-span bridges and tall buildings. Also Monte Carlo simulation often serves as a benchmark to evaluate accuracy of other approaches such as the frequency domain based approaches.

One of most important tasks in Monte Carlo simulation is to generate sample functions of stochastic excitations, such as ground motions, wind and wave fluctuations. Based on spectral representation theorem, the spectral representation method (SRM) is widely used in the simulation of sample functions of stochastic processes and fields due to its expediency (e.g., [29,30,24,17]). The computational efficiency of SRM-based simulation of stationary Gaussian processes and fields can be considerably enhanced by the way of introducing the fast Fourier transform (FFT) [36]. Deodatis [11] summarized the classical SRM-based simulation for

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multivariate stationary Gaussian processes and Shinozuka and Deodatis [32] reviewed the simulation of stationary Gaussian fields by SRM.

Despite the successful application of SRM in generating sample functions for stationary Gaussian processes, there is room to improve its efficiency, especially for large structures such as tall buildings and long-span bridges where a very large number of simulation points could be required. One example is the wind-induced buffeting analysis of large-span bridges, where hundreds of wind velocity fluctuations need to be generated to better represent the wind velocity field along the length of the deck. Also, perhaps around a thousand wind velocity locations are needed for the analysis of wind–vehicle–bridge systems when a high-speed train passes a bridge. For the bridge built over a homogeneous terrain such as coastal area or flat plain, the wind velocity field can be modeled as a multivariate homogeneous random process, i.e., the auto power spectral densities (PSDs) at evenly-spaced simulation points are same and the cross PSD is a function of separation distance between two simulation points. In some applications, additional simulation points need to be included to a set of uniformly distributed points in order to make the simulated wind velocity field consistent to the need of structural dynamic analysis.

The simulation of aforementioned multivariate processes requires significant computational time and storage memory if the classical SRM is adopted. For instance, the double-index frequency technique introduced by Deodatis [11] can generate ergodic sample functions. However, this treatment needs decomposition of a very large spectral matrix when the number of simulation points is large. Besides, the generated ergodic sample will have a very long period since the period of samples has a linear relation with the number of the simulation points, which reduces the simulation efficiency and increases computational demand for the ensuing response analysis. To enhance the simulation efficiency or save the memory requirements in the generation of random processes, the common practice is to reduce the computational demand on the Cholesky decomposition of spectral matrix in that the decomposition of large matrix consumes significant computer resources. Yang et al. [38] and Cao et al. [4] proposed closed-form formula for wind field simulation to avoid Cholesky decomposition of the spectral matrix. This method is effective only if the coherence function of the random process follows a real exponential function. However, many studies show that this simple model for winds may not be appropriate, and several more meaningful non-exponential models have been introduced (e.g., [22,23,12,27]). In addition, the closed-form formula technique is only valid for the multivariate homogeneous random process with evenly distributed simulation points. Ding et al. [15] derived an efficient scheme to decrease the computational amount of spectral matrix decomposition, where the decomposition of the spectral matrix is performed on a set of single-index frequencies. Huang et al. [18] developed a new form of Cholesky decomposition to improve the decomposition efficiency of the spectral matrix by separating the phase from the spectral matrix. Apart from the widely-used Cholesky decomposition, the proper orthogonal decomposition (POD), or eigenvector decomposition has been used in the simulation of multicorrelated stationary random processes or fields in both the time and frequency domains (e.g., [24,37,14,7,6,26]). In addition, POD has also been widely employed to simplify the simulation of wind velocity fields such as random wind fields with varying PSDs at different locations, complex wind fields with three correlated turbulence components along a linear domain and complex wind fields considering three turbulent components in a spatial domain, where the closed-form solution for the eigenvalue and eigenfunction of the principal turbulence was derived for exponential coherence function [34,5,8]. The core of utilizing POD is that the simulation may be carried out on only a few lower modes which contain most of the energy. In this case, the simulation efficiency can be improved. However, the truncation of higher modes may cause noticeable errors if local structural response is of interest [6]. Besides, POD may fall short in very broad-band processes or fields such as the white noise. It should be noted that both the Cholesky and POD decompositions can be unified under the concept of stochastic decomposition approach proposed by Li and Kareem [24,25].

On the other hand, the application of FFT is a good option to improve the simulation efficiency for wind velocity processes. As is well known, the two-dimensional (2D) FFT algorithm can significantly expedite the simulation of Gaussian stationary random field (e.g., [39,32]). Hence, if the multivariate wind velocity process can be transformed to a space–time random field, 2D FFT algorithm could be introduced and simulation efficiency may be significantly enhanced. Although the space-time random field has been applied to simulate stochastic excitations such as seismic ground motions and wind fields (e.g., [28,40,3]), a systematic treatment of the subject has thus far not been addressed.

To efficiently simulate the wind velocity field along the bridge deck with a large number of points, a hybrid simulation approach will be developed in this study. Primarily, a multivariate homogeneous random process associated with a set of uniformly distributed simulation points can be simulated via the random field-based simulation method. Then, the wind velocities at a few unevenly distributed points can be stochastically interpolated from the simulated wind velocities. Among many interpolation schemes, the POD is a powerful one which has been widely used in stochastic mechanics and wind engineering including the compression and reconstruction of wind pressures (e.g., [1,33,6]) and nonstationary process simulation [20]. This study utilizes a POD-based interpolation scheme.

This paper is organized as follows. First, the characteristics of the wind velocity field with a large number of simulation points are discussed briefly. Then an efficient simulation approach relying on the random field-based SRM and POD-based interpolation is introduced to generate the sample function of this wind velocity field. Especially, the applicability, accuracy, ergodicity and efficiency of the random field-based SRM are examined. Furthermore, a numerical example is used to demonstrate the effectiveness of the proposed approach. In the end, some concluding remarks will be given.

2. Wind velocity field

Let $[p_1^0(t), p_2^0(t), \ldots, p_l^0(t)]^T$ be a zero-mean l-variate wind velocity field along a bridge deck over a homogeneous terrain. The underlying correlation function and associated spectral matrices are given by

$$\mathbf{R}^0(\tau) = \begin{bmatrix} R_{11}^0(\tau) & R_{12}^0(\tau) & \cdots & R_{1l}^0(\tau) \\ R_{21}^0(\tau) & R_{22}^0(\tau) & \cdots & R_{2l}^0(\tau) \\ \vdots & \vdots & \ddots & \vdots \\ R_{l1}^0(\tau) & R_{l2}^0(\tau) & \cdots & R_{ll}^0(\tau) \end{bmatrix} \tag{1}$$

$$\mathbf{S}^0(\omega) = \begin{bmatrix} S_{11}^0(\omega) & S_{12}^0(\omega) & \cdots & S_{1l}^0(\omega) \\ S_{21}^0(\omega) & S_{22}^0(\omega) & \cdots & S_{2l}^0(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{l1}^0(\omega) & S_{l2}^0(\omega) & \cdots & S_{ll}^0(\omega) \end{bmatrix} \tag{2}$$

where τ and ω are the time lag and circular frequency, respectively; $R_{jk}^0(\tau)$ and $S_{jk}^0(\omega)$ ($j, k = 1, 2, \ldots, l$) are the cross correlation