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# Dynamic analysis of stochastic structural systems using frequency adaptive spectral functions

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ABSTRACT

A novel Galerkin subspace projection scheme for linear structural dynamic systems with stochastic coefficients is developed in this paper. The fundamental idea is to solve a discretized stochastic system in the frequency domain by projecting the solution on a reduced subspace of eigenvectors of the deterministic operator weighted by a set of frequency dependent stochastic functions, termed as the spectral functions. These spectral functions are rational functions of the input random variables and a study of the different orders of spectral functions are presented. A set of undetermined Galerkin coefficients are utilized to orthogonalize the residual to the reduced eigenvector space in the mean sense. The complex system response is represented explicitly with these Galerkin coefficients in conjunction with the modal basis and the associated stochastic spectral functions. The statistical moments of the solution are evaluated at all frequencies and the solution accuracy is verified in terms of a relative error norm. Two examples involving a beam and a plate with stochastic parameters subjected to harmonic excitations have been studied. The results are compared with the direct Monte-Carlo simulation, the classical Neumann expansion technique and the polynomial chaos method for different orders stochastic functions and varying degrees of variability of input randomness.

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## 1. Introduction

Computational models of physical systems often use idealized approximations, such as in parameter values and boundary conditions, which cannot be known for certain. This has led to investigation into stochastic computer models which incorporate probabilistic description of the uncertain quantities into the model (such as [1–3]). In this study we concentrate on the frequency domain response of damped structural dynamic systems with parametric uncertainty which is multiplicative in nature. The objective is to propose a novel uncertainty propagation scheme for probabilistic inputs to the structural dynamic model (as parametric uncertainty) and investigate the accuracy, convergence and computational cost of the proposed method. There are two broad classes of the uncertainty propagation techniques for the stochastic systems – statistical sample based simulations and the non-statistical analytical methods.

Various Monte Carlo Simulation (MCS) techniques belong to the class of non-intrusive sample based methods and have been used in context of the structural dynamics problems [4]. The slow convergence rate of MCS makes it unfeasible for practical implementation and various improved sampling techniques such as the importance sampling, Latin hypercube sampling, have been proposed which can be regarded as the “variance reduction techniques” [5]. The limitations of these techniques are dictated by the dimension of the stochastic space. Uncertain structural systems represented by few random variables subjected to deterministic loading can be well-suited for such variance reduction procedures [6].

The alternatives to the sampling techniques provide an explicit functional relationship between the input random variables which allows easy evaluation of the functional statistics. Such solvers include perturbation method [7], or equivalently the lower-order Taylor approximation and the Neumann expansion method, [8,9] all of which comes down to the estimation of response surface in a parameter space. On the other hand the stochastic Galerkin methods [10–12] allows the response to be expressed with orthogonal basis functions spanning the stochastic space. The accurate estimation of the high order statistical moments, or the full probability distribution function, of the response requires high degree polynomials, which in turn results in high dimensional block sparse linear system of equations. Krylov-type iterative techniques have been proposed to solve such systems efficiently by exploiting the sparsity of the system [13,14]. However, the difficulty to build efficient preconditioners and the memory

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requirements induced by these techniques are still challenging and active areas of research.

We consider a bounded domain $\mathcal{D} \in \mathbb{R}^d$ with piecewise Lipschitz boundary $\partial\mathcal{D}$, $d \le 3$ is the spatial dimension and $t \in \mathbb{R}^+$ is the time. We take $(\Theta, \mathcal{F}, P)$ as the probability space where $\theta \in \Theta$ is a sample point from the sampling space $\Theta$, $\mathcal{F}$ is the complete Borel $\sigma$-algebra over the subsets of $\Theta$ and $P$ is the probability measure. We consider here the linear stochastic partial differential equation (pde) for elastodynamic systems with parametric uncertainty:

$$\rho(\mathbf{r}, \theta)\frac{\partial^2 u(\mathbf{r}, t, \theta)}{\partial t^2} + \mathfrak{L}_{\mathbf{c}}\frac{\partial u(\mathbf{r}, t, \theta)}{\partial t} + div(\sigma_\alpha(u(\mathbf{r}, t, \theta))) = p(\mathbf{r}, t); \quad \mathbf{r} \in \mathcal{D}, \ t \in [0, T], \ \theta \in \Theta \tag{1}$$

with the associated Dirichlet condition $u(\mathbf{r}, t, \theta) = 0$; $\mathbf{r}$ on $\partial\mathcal{D}$. Here $\sigma_\alpha(u(\mathbf{r}, t, \theta))$ is the stress tensor with stiffness coefficient $\alpha(\mathbf{r}, \theta)$ modeled as a stationary, square integrable random field such that $\alpha: \mathbb{R}^d \times \Theta \to \mathbb{R}$. The operator $div(\sigma_\alpha)$ is taken to be a self-adjoint stochastic stiffness operator. $\mathfrak{L}_{\mathbf{c}}$ is the damping operator containing the stochastic coefficient vector $\mathbf{c}(\mathbf{r}, \theta)$. The damping operator along with its coefficients can be utilized to represent various damping models like the strain rate dependent viscous damping or the velocity dependent viscous damping. $p(\mathbf{r}, t)$ is the deterministic excitation field for which the solution $u(\mathbf{r}, t, \theta)$ is sought. To perform harmonic analysis, Eq. (1) is transformed to the frequency domain as

$$-\omega^2\rho(\mathbf{r}, \theta)\tilde{u}(\mathbf{r}, \omega, \theta) + i\omega\mathfrak{L}_{\mathbf{c}}\tilde{u}(\mathbf{r}, \omega, \theta) + div\left(\sigma_\alpha(\tilde{u}(\mathbf{r}, \omega, \theta))\right) = \tilde{p}(\mathbf{r}, \omega); \quad \omega \in \Omega \tag{2}$$

where $\Omega$ denotes the frequency space of the problem, with $\tilde{p}$ and $\tilde{u}$ representing the complex harmonic amplitudes. $\mathbf{E}(\alpha)$ in the stress–strain relationship $\sigma_\alpha = \mathbf{E}(\alpha)$: $\varepsilon$ is the symmetric positive definite elasticity tensor which depends on the scalar random parameter $\alpha$ with $\varepsilon$ being the strain tensor expressed as $\varepsilon = \mathbf{D}\tilde{\mathbf{u}}$. Well established techniques of variational formulation of the displacement-based deterministic finite-element methods [7,15] gives the following bilinear form for the elastodynamic system:

$$\mathcal{B}(\tilde{\mathbf{v}}, \tilde{\mathbf{u}}; \theta) = -\omega^2\int_{\mathcal{D}}\tilde{\mathbf{v}}\rho(\mathbf{r}, \theta)\tilde{\mathbf{u}}\, d\mathcal{D} + i\omega\int_{\mathcal{D}}\tilde{\mathbf{v}}\mathfrak{L}_{\mathbf{c}}\tilde{\mathbf{u}}\, d\mathcal{D} + \int_{\mathcal{D}}\{\mathbf{D}\tilde{\mathbf{v}}\}^T\mathbf{E}(\alpha)\left\{\mathbf{D}\tilde{\mathbf{u}}\right\} d\mathcal{D} \tag{3}$$

$$\mathcal{L}(\tilde{\mathbf{v}}; \theta) = \int_{\mathcal{D}}\tilde{\mathbf{v}}\tilde{p}\, d\mathcal{D}$$

$$\text{so that,} \quad \mathcal{B}(\tilde{\mathbf{v}}, \tilde{\mathbf{u}}; \theta) = \mathcal{L}(\tilde{\mathbf{v}}; \theta) \qquad \forall\ \tilde{\mathbf{v}} \in \mathcal{E}[\mathcal{D}] \tag{4}$$

where $\mathcal{E}[\mathcal{D}]$ is the space of admissible trial functions which have finite strain energy on the spatial domain and satisfying the prescribed boundary conditions. Eq. (4) gives a set of discretized linear algebraic equations in terms of the mass, damping and stiffness matrices. These can be expressed in a compact form as

$$\mathbf{A}(\omega, \theta)\tilde{\mathbf{u}}(\omega, \theta) = \tilde{\mathbf{p}}(\omega); \quad \forall\ \theta \in \Theta;\ \omega \in \Omega;\ \mathbf{A} \in \mathbb{C}^{n\times n};\ \tilde{\mathbf{u}}, \tilde{\mathbf{p}} \in \mathbb{C}^n \tag{5}$$

where $\mathbf{A}(\omega, \theta)$ is the complex frequency dependent coefficient matrix which inherits the uncertainty of the random parameters involved in the governing pde. The detailed description of these matrices is given in Section 3.1.

The paper has been arranged as follows. In Section 2 a brief overview of the stochastic finite element method is presented. The spectral decomposition technique in space of eigen vectors adopted in this study is detailed in Section 3 along with a description of the spectral functions proposed here. In Section 4 a reduced Galerkin error minimization approach is proposed. The post-processing of the results to obtain the response moments are discussed in Section 5. Based on the theoretical results, a simple computational approach is shown in Section 6 where the proposed method of reduced spectral basis is applied to the stochastic dynamical system of an one-dimensional Euler–Bernoulli beam and a two-dimensional Kirchhoff–Love thin plate. A summary of the results and major conclusions arising from this study are given in Section 7.

## 2. Brief review of the stochastic finite element method

### 2.1. Discretization of random fields

The parametric uncertainty is generally associated with a covariance function $cov[a]: \mathcal{D} \times \mathcal{D} \to \mathbb{R}$ defined on the open, bounded polygonal domain in $\mathcal{D}$. For second order random fields, there is a compact self-adjoint operator:

$$T_a v(\cdot) = \int_{\mathcal{D}} cov[a](\mathbf{r}, \cdot)v(\mathbf{r})d\mathbf{r} \quad \forall\ v \in L^2(\mathcal{D}) \tag{6}$$

along with a sequence of non-negative eigenpairs $\{(\lambda_i, \varphi_i)\}_{i=1}^{\infty}$ which describes the eigenvalue problem as

$$T_a\varphi_i = \lambda_i\varphi_i, \quad (\varphi_i, \varphi_j)_{L^2(\mathcal{D})} = \delta_{ij} \tag{7}$$

The truncated Karhunen–Loève (KL) expansion of the stochastic process $a(\mathbf{r}, \theta)$ using these eigen-functions is expressed as

$$\hat{a}_m(\theta, \mathbf{r}) = E[a](\mathbf{r}) + \sum_{i=1}^{m}\sqrt{\lambda_i}\varphi_i(\mathbf{r})\xi_i(\theta) \quad \forall\ m \in \mathbb{N}_+ \tag{8}$$

where $E[a](\mathbf{r})$ is the mean function, $\{\xi_i(\theta)\}_{i=1}^{m}$ are a set of mutually independent, uncorrelated standard Gaussian random variables with zero mean ($E(\xi_i) = 0$) and unit variance ($E(\xi_i^2) = 1$). The eigenfunctions $\varphi_i(\mathbf{r})$ can be assumed to have sufficient smoothness for smooth covariance functions, and if the eigenpairs are decaying according to at least $\sqrt{\lambda_k}\ \|\varphi_k\|_{L^\infty(\mathcal{D})} = O(1/(1 + k^s))$ for some decay exponent $s > 1$, then $\|a - \hat{a}_m\|_{L^\infty(\mathcal{D})} \to 0$, [11]. For practical engineering problems, the parametric randomness is modeled with a finite set of random variables $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_m): \Theta \to \mathbb{R}^m$, using first few largest eigenpairs in the reduced probability space [16].

For arbitrary random field models, the random parameter can be expressed in a mean-square convergent series using a finite order chaos-expansion in terms of the basic independent identically distributed (iid) random variables $\hat{\boldsymbol{\xi}}(\theta) = \{\hat{\xi}^{(1)}, \ldots, \hat{\xi}^{(n)}\}$ as $a(\mathbf{r}, \theta) = \sum_{i=0}^{p}\mathcal{H}_i(\hat{\boldsymbol{\xi}}(\theta))a_i(\mathbf{r})$ where $\mathcal{H}_i(\hat{\boldsymbol{\xi}}(\theta))$ are the multivariate orthogonal polynomial functions depending on the joint probability density function of the stochastic Hilbert space. The solution methodology presented in this paper is applicable to this kind of general decomposition of the random field.

### 2.2. Spectral methods and other solution techniques

The spatially discretized solution vector $\tilde{\mathbf{u}}(\omega, \theta)$ lies in the tensor product space $\mathbb{C}^n \otimes \Upsilon$, where $\Upsilon$ is an ad-hoc function space for real-valued random variables [10]. Since the stochastic system is represented with finite set of iid random variables $\hat{\boldsymbol{\xi}}(\theta) = \{\hat{\xi}^{(1)}, \ldots, \hat{\xi}^{(p)}\}$ as in Section 2.1, the stochastic space is given as $\Upsilon_p \subset \Upsilon$. Given that each random component $\hat{\xi}^{(i)}$ is independent, $\Upsilon_p$ is of the tensor product form $\Upsilon^1 \otimes \Upsilon^2 \otimes \ldots \otimes \Upsilon^p$. The solution vector in Eq. (5) can be expressed in the form:

$$\tilde{\mathbf{u}}(\omega, \theta) = \sum_{\alpha \in \mathcal{I}_p}\mathcal{H}_\alpha(\omega, \theta)\tilde{u}_\alpha(\omega); \quad \tilde{u}_\alpha(\omega) \in \mathbb{C}^n \tag{9}$$

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