Probabilistic Engineering Mechanics 39 (2015) 39–45

Contents lists available at ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

Cumulative PSO-Kriging model for slope reliability analysis

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ARTICLE INFO

Article history:
Received 24 August 2014
Received in revised form
15 December 2014
Accepted 16 December 2014
Available online 18 December 2014

Keywords:
Reliability analysis
Kriging model
Particle swarm optimization
Cumulative sampling
Breakwater

ABSTRACT

The particle swarm optimization (PSO) algorithm is introduced in the Kriging modeling process to overcome the limits of pattern search method's single-point search scheme as well as its heavy dependence on the initial guess solution when obtaining the optimal correlation parameters. PSO-Kriging is proved to give better simulation by interpolating and extrapolating the unobserved points. In reliability analysis, cumulative formation of repeatedly using sampling points in previous iterations is introduced into PSO-Kriging and the classic response surface method (RSM). Cumulative formation can take full advantage of available sampling information and avoid reciprocating oscillation in the iterative process. One explicit nonlinear limit state function example demonstrated that cumulative scheme can make both PSO-Kriging and RSM much more effective, no matter latin hypercube sampling (LHS) or iteratively interpolating sampling (IIS) approach is utilized. Cumulative PSO-Kriging seems to be even more stable and efficient. Two slope reliability analysis examples including a practical nuclear plant breakwater's reliability analysis problem proved that the proposed cumulative PSO-Kriging is very suitable for the reliability analysis of real engineering structures.

1. Introduction

Uncertainties are observed in material and geometric properties as well as external loads during structures' lifetime. Therefore, recent decades are characterized by growing realization of the fact that, for meaningful analysis of engineering structures, uncertainty should be taken into account and the structural reliability theory provides a good methodology to do this [1]. The need is more strong in geotechnical engineering, because natural materials, from which dams, retaining walls and breakwaters are made, exhibit more and larger uncertainties than in structural engineering obviously. Duncan [2] has pointed out that the reliability analysis offers a useful supplement to conventional stability analyses, because the resultant reliability index contains more information than the deterministic safety factor.

In reliability analysis, the performance of a structure or any other system is governed by a limit state function (LSF) or performance function $g(\mathbf{x})$ and $\mathbf{x} = [x_1, \ x_2, \ \cdots \ x_n]^T$ is the random variable vector, representing uncertain quantities. $g(\mathbf{x}) \le 0$ denotes the failure domain. The probability of failure is expressed as a multi-dimensional integral of the joint probability density function over the failure domain. Difficulty in computing this multi-dimensional integral has led to various approximation methods. The first order reliability method (FORM) obtains the approximate solution by the linear expansion of $g(\mathbf{x})=0$ at the design point $\mathbf{x}^*$, which is the most likely failure point. The second order reliability method (SORM) obtains the approximate solution by expanding $g(\mathbf{x})=0$ at $\mathbf{x}^*$ with the second order effects (the curvatures) considered.

In practice, $g(\mathbf{x})$ is usually implicit and must be solved using structural/mechanical analysis. For example, in geotechnical engineering, the limit state function of slope stability is usually defined as $g(\mathbf{x})=F_s(\mathbf{x})-1.0$, where $F_s(\mathbf{x})$ is the factor of safety calculated by the limit equilibrium method (LEM) or the strength reduction method (SRM) based on the finite element method (FEM)/finite difference method (FDM). Here $F_s(\mathbf{x})$ is implicit and then $g(\mathbf{x})$ is implicit, which requires a considerable amount of computation cost for every analysis. In this case, FORM/SORM are not directly applicable and Monte Carlo simulation (MCS) seems to be suitable. The advantages of MCS are obvious, but an innate disadvantage of it is the formidable computational effort with tens to hundreds of thousands sampling cycles. To overcome this disadvantage, numerous variance reduction techniques have been proposed, such as importance sampling [3] and directional simulation [4]. However, simulation methods even with importance sampling schemes are time-consuming and impractical to problems involving low probability of failure or the problems that require a considerable amount of computation in each sampling cycle, such as slope reliability analysis in this paper.

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http://dx.doi.org/10.1016/j.probengmech.2014.12.001
0266-8920/

Another kind of methodologies, which may not be as accurate as the above methods but is nevertheless feasible for a wider spectrum of problem, is the surrogate model methods. In the surrogate model methods, an explicit function intended to substitute the implicit performance function is fitted through planed or random samples and the failure probability of the explicit function replace that of the actual implicit performance function. Response surface method (RSM) [5] and Kriging model [6] are such methods.

In RSM, a polynomial function is usually used as a surrogate model to approximate the unknown implicit performance function. Xu and Low [7] utilized RSM combined with FEM to calculate the reliability index of slopes and demonstrated the feasibility of using surrogate models in slope reliability analysis. However, Kaymaz and McMahon [8] reported that conventional RSM required large computational efforts and showed loss of accuracy in the cases of problems exhibiting acute nonlinearity.

The Kriging model is an alternative surrogate model that was introduced by Sacks et al. [6] and is mainly used to solve deterministic optimization problems [9]. Recently, the Kriging model was applied to structural reliability analysis [10]. Kaymaz [10] computed the reliability index for assumed performance functions rather than real structures using DACE (design and analysis of computer experiments) – a Kriging MATLAB toolbox developed by Lophaven [11]. Gaspar et al. [12] pointed out that the Kriging model can provide significantly more accurate failure probability predictions as compared with the polynomial regression models. Their interpolation capability and flexibility or local adaptability have shown to be important features for structural reliability analysis problems.

When constructing a Kriging model, one should first determine values of some correlation parameters, $\boldsymbol{\theta}$. Their values have great effect on the performance of the Kriging model [10]. In DACE, the pattern search method was utilized to find the optimal values of $\boldsymbol{\theta}$. However, this method tends to become trapped in local minima, and the solution is greatly affected by the given initial value [13]. Therefore, Luo et al. [13] used the artificial bee colony algorithm to optimize $\boldsymbol{\theta}$ and You and Jia [14] utilized the genetic algorithm.

In the present study, particle swarm optimization (PSO) algorithm is introduced into Kriging modeling process to guarantee the optimal correlation parameters under any initial conditions. A typical example is given to validate the good curve fitting performance of PSO-Kriging model. In reliability analysis, cumulative formation of repeatedly using sampling points obtained in previous iterations is introduced into PSO-Kriging. One explicit nonlinear limit-state function example is selected to validate the accuracy and efficiency of the cumulative PSO-Kriging. Finally, the algorithm is employed in slope reliability analyses, including reliability analysis of a nuclear power plant breakwater.

2. Adaption of Kriging model

2.1. Kriging theory

Kriging is a statistical surrogate model that can be used to approximate the input-output function on its input space based on a small initial design of experiments (DOE). The Kriging model consists of a Gaussian process that can be expressed, for any input vector $\mathbf{x} \in \Re^n$, as

$$\hat{y}(\mathbf{x}) = m(\boldsymbol{\beta}, \mathbf{x}) + z(\mathbf{x}) \tag{1}$$

where $m(\bullet)$ is an optional regression model estimated from available data and expressed as a linear combination of p chosen polynomial functions $f_j(\bullet)$ in DACE,

$$m(\boldsymbol{\beta}, \mathbf{x}) = \sum_{j=1}^{p} \beta_j f_j(\mathbf{x}) = \mathbf{f}^T(\mathbf{x})\boldsymbol{\beta} \tag{2}$$

The coefficients $\boldsymbol{\beta}=[\beta_1, \ldots, \beta_p]^T$ are regression parameters. The random process $z(\bullet)$ is the stochastic error and assumed to have zero mean and covariance between $z(\boldsymbol{\omega})$ and $z(\mathbf{x})$ as follows,

$$\mathrm{cov}(z(\boldsymbol{\omega}), z(\mathbf{x})) = \sigma^2 R(\boldsymbol{\omega}, \mathbf{x}) \tag{3}$$

where σ^2 is the process variance and $R(\bullet,\bullet)$ is a parametric correlation function. Like $\mathbf{x}$, $\boldsymbol{\omega}$ is also an input vector and $\boldsymbol{\omega} \in \Re^n$. Several correlation functions are available in the literature. Kaymaz [10] tested three types of correlation functions, i.e., exponential, linear and Gauss and concluded that the Gauss correlation function is more suitable for the problems having nonlinear limit state functions. In this study, the Gaussian correlation function is adopted,

$$R(\boldsymbol{\omega}, \mathbf{x}) = \exp\left(-\sum_{i=1}^{n} \theta_i(\omega_i - x_i)^2\right) \tag{4}$$

where θ_i are scale factors whose estimation is described in the following.

Suppose $\mathbf{S}=[\mathbf{s}_1, \ldots, \mathbf{s}_m]^T$ is a set of m design sites and $\mathbf{Y}=[y_1, \ldots, y_m]$ is the corresponding response set. Define $\mathbf{R} = (R_{kl})_{m\times m}$ as the correlation matrix of stochastic-process between z's at design sites $\mathbf{S}$ and $R_{kl} = R(\mathbf{s}_k, \mathbf{s}_l)$, $k, l = 1, \cdots, m$. Further define $\mathbf{F} = \left(F_{kj}\right)_{m\times p}$ and $F_{kj}=f_j(\mathbf{s}_k)$. For any given $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_n)$, the generalized least square solutions for $\boldsymbol{\beta} = \left(\beta_1, \cdots, \beta_p\right)$ and σ^2 are as follows.

$$\hat{\boldsymbol{\beta}} = (\mathbf{F}^T\mathbf{R}^{-1}\mathbf{F})^{-1}\mathbf{F}^T\mathbf{R}^{-1}\mathbf{Y} \tag{5}$$

$$\hat{\sigma}^2 = \frac{1}{m}\left(\mathbf{Y} - \mathbf{F}\hat{\boldsymbol{\beta}}\right)^T\mathbf{R}^{-1}\left(\mathbf{Y} - \mathbf{F}\hat{\boldsymbol{\beta}}\right) \tag{6}$$

As a Gaussian process, the optimal coefficients $\boldsymbol{\theta}^*$ of the correlation function solves

$$\min_{\boldsymbol{\theta}}\left\{\varphi(\boldsymbol{\theta}) \equiv |\mathbf{R}|^{\frac{1}{m}}\sigma^2\right\} \tag{7}$$

where $|\mathbf{R}|$ is the determinant of $\mathbf{R}$. This definition of $\boldsymbol{\theta}^*$ corresponds to maximum likelihood estimation [15]. Once $\boldsymbol{\theta}^*$ is obtained, the Kriging surrogate model based on m design sites $\mathbf{S}$ is as follows,

$$\hat{y}(\mathbf{x}) = \mathbf{f}^T(\mathbf{x})\hat{\boldsymbol{\beta}} + \mathbf{r}^T(\mathbf{x})\mathbf{R}^{-1}\left(\mathbf{Y} - \mathbf{F}\hat{\boldsymbol{\beta}}\right) \tag{8}$$

where $\mathbf{r}(\mathbf{x}) = [R(\mathbf{x}, \mathbf{s}_1), \cdots, R(\mathbf{x}, \mathbf{s}_m)]^T$ is the correlation matrix of stochastic-process between z's at design sites $\mathbf{S}$ and the untried point $\mathbf{x}$.

It has been proved that if the optimal correlation parameters $\boldsymbol{\theta}^*$ in the maximum likelihood sense can be guaranteed under any initial conditions, the optimal unbiased characteristic for the Kriging prediction can also be assured [11]. In DACE, the pattern search method was used to find $\boldsymbol{\theta}^*$. However, this method could not guarantee to find the optimal values of $\boldsymbol{\theta}$ under any initial conditions, which will affect the performance of the Kriging model [13]. Therefore, Luo et al. [13] used the artificial bee colony algorithm to optimize $\boldsymbol{\theta}$ and You and Jia [14] utilized the genetic algorithm. In this study, PSO, an efficient global optimization algorithm, was used to search $\boldsymbol{\theta}^*$.

2.2. PSO

Particle swarm optimization (PSO), which is based on a social-psychological model, is one of the modern heuristic algorithms for optimization [16]. In PSO, the so-called swarm is composed of a set of particles, which move in the search space of an optimization