Available online at www.sciencedirect.com
ScienceDirect
Journal of Hydrodynamics
2017,29(6):1081-1084
DOI: 10.1016/S1001-6058(16)60821-1
www.sciencedirect.com/
science/journal/l0016058

Two timescales for longitudinal dispersion in a laminar open-channel flow *

Yu-fei Wang (王宇飞), Wen-xin Huai (槐文信), Zhong-hua Yang (杨中华), Bin Ji (季斌)
State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan 430072, China, E-mail: yufeiwang@whu.edu.cn

(Received June 13, 2017, Revised July 14, 2017)

Abstract: At small dimensionless timescales $T(=tD/H^2)$, where t is the time, H is the depth of the channel and D is the molecular diffusion coefficient, the mean transverse concentration along the longitudinal direction is not in a Gaussian distribution and the transverse concentration distribution is nonuniform. However, previous studies found different dimensionless timescales in the early stage, which is not verified experimentally due to the demanding experimental requirements. In this letter, a stochastic method is employed to simulate the early stage of the longitudinal transport when the Peclet number is large. It is shown that the timescale for the transverse distribution to approach uniformity is $T=0.5$, which is also the timescale for the dimensionless temporal longitudinal dispersion coefficient to reach its asymptotic value, the timescale for the longitudinal distribution to approach a Gaussian distribution is $T=1.0$, which is also the timescale for the dimensionless history mean longitudinal dispersion coefficient to reach its asymptotic value.

Key words: Early stage, longitudinal dispersion, random walk particle method, scalar transport

The longitudinal dispersion is an important issue in studying the scalar transport in flow systems (e.g., the open-channel flow and the pipe flow)[1,2]. The longitudinal dispersions share the same behavior although the cross sections of the flow systems may be different (e.g., rectangular and circular sections), thus it is helpful to study the longitudinal dispersion in the laminar open-channel flow. When the time scale is large, the longitudinal dispersion coefficient, which reflects the spreading rate in the longitudinal direction, can be solved easily. Using some reasonable assumptions (e.g., the longitudinal diffusion can be omitted when the Peclet number (Pe) is large), Chatwin and Sullivan[1] found that the dispersion coefficient for the laminar channel flow can be expressed as $k=2H^2U^2/(105D_z)$, where H is the channel depth, U is the mean velocity and D_z the transverse diffusion coefficient. However, the formula given by Chatwin and Sullivan[1] can only be used at a large timescale when the transverse concentration distribution is uniform and the longitudinal concentration is in a Gaussian distribution. This is because in the early stage, the actual problems are as follows: (1) the transverse concentration distribution is nonuniform, (2) the longitudinal dispersion coefficient is not a constant value and (3) the longitudinal mean transverse concentration is not in a Gaussian distribution, with absolute skewness $\lambda(=\nu_3^2/\nu_2^3)$ being not zero, where ν_n is the nth integral central moment of c

$$\nu_n(t)=\frac{\iiint\limits_{\text{whole area}}(x-x_g)^n c(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z}{\iiint\limits_{\text{whole area}}c(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z} \tag{1}$$

where x_g is the location of the centre of gravity. Thus, the initial stage of the scalar transport was widely studied[3-5]. However, the timescales vary from one study to another, and without experiments to verify-their results due to the demanding experimental conditions. One might be confused in the face of many conflict results. Therefore, the study of the dimensionless timescales could offer an alternative simple stan-

* Project supported by the National Natural Science Foundation of China (Grant Nos. 51439007, 11372232).
Biography: Yu-fei Wang (1989-), Male, Ph. D. Candidate
Corresponding author: Wen-xin Huai,
E-mail: wxhuai@whu.edu.cn

dard to evaluate the previous theoretical studies.

In this letter, we concentrate on the developments of the transverse uniformity index (UI), the dimensionless history mean longitudinal dispersion coefficient (K), the dimensionless temporal longitudinal dispersion coefficient (K_t) and the absolute skewness of longitudinal concentration distribution (λ), without touching other indices, such as the peak concentration and the length of the solute.

The random walk particle method (RWPM) is used here. We regard the concentration as a large number of independent particles[6]. Each particle's position can be updated by an advection displacement (caused by the local velocity) superposed by a random displacement. Thus, after τ, the new position of the particle at $\boldsymbol{X}_t$, is given as follows

$$\boldsymbol{X}_{t+\tau} = \boldsymbol{X}_t + \boldsymbol{u}\tau + P(2D\tau)^{1/2} \quad (2)$$

where $\boldsymbol{u}$ is $(u,0)^t$ for one-dimensional laminar flow, $\boldsymbol{D}$ is the diffusion matrix and $\boldsymbol{P}$ is the probability matrix. Here, the scalar transport is simulated for $Pe = 100$, which is common in practice. We consider 1 000 particles distributed uniformly in the transverse direction, from $z = 0$ m to $z = 2.5\times 10^{-4}$ m. The time step is 10^{-4} s and $D_x = D_z = 10^{-9}\ \mathrm{m^2/s}$.

In most of the previous studies, the transverse concentration distribution is assumed as uniform to solve the ADE[1]. Chatwin[3] suggested that this assumption is right provided $T > 0.25$, whereas the results of Bailey and Gogarty[4] and Wu and Chen[5] showed that the time to approach the transverse uniformity is $T > 0.69$ and $T > 10$ ($T = tD/R^2$ for a pipe flow, where R is the radius), respectively.

Here, the uniformities in three typical domains are analysed. The three domains are $p_c \in (x_g - 0.1\sigma, x_g + 0.1\sigma)$, $p_u \in (x_g - 1.1\sigma, x_g - 0.9\sigma)$ and $p_d \in (x_g + 0.9\sigma, x_g + 1.1\sigma)$ where $\sigma = \nu_2^{1/2}$. The transverse uniformity index (UI) is defined as $UI = \sigma_l / \bar{l}$, where $\sigma_l = \sqrt{N_t^{-1}\sum_{i=1}^{N_t}(l_i - \bar{l})^2}$ is the standard deviation of the length that each particle occupies, N_t is the number of the particles in the region, l_i is the length that the ith particle occupies and $\bar{l}(= H/N_t)$ is the mean value of l_i. l_i can be obtained as:

$$l_1 = \frac{z_1 + z_2}{2},\ l_N = \frac{H - (z_{N_t - 1} + z_{N_t})}{2},$$

$$l_i = \frac{z_{i+1} - z_{i-1}}{2},\ i = 2,3,\cdots,N-1 \quad (3)$$

where $z_1 < z_2 < \cdots < z_{N_t}$. For an ideal uniformity distribution, the value of UI should be zero (i.e., $l_i = \bar{l}$, $i = 1,2,\cdots$). However,it is not zero in practice because of the randomness of the molecular transport.

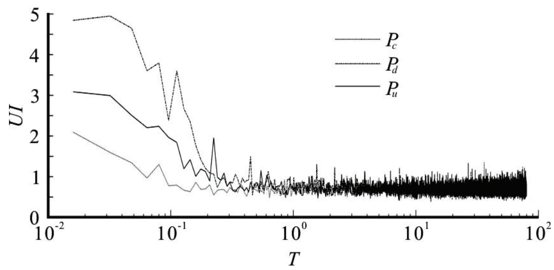

Fig.1 Development of transverse uniformity index (UI)

As can be seen from Fig.1, the values of UI in three domains decrease and approach the asymptotic value of 0.7 at around $T = 0.5$, i.e., the time to approach the transverse uniformity is $T = 0.5$.

Besides, the longitudinal dispersion coefficient is time dependent in the early stage. Thus, some unreasonable explanations[7] were given to describe the results at the near-source region or in the early stage. Gill and Sankarasubramanian[8] shows that the longitudinal dispersion coefficient increases with time and reaches its asymptotic value at $T = 0.1$ - 0.2. The numerical result of Gill and Sankarasubramanian[9] shows that the longitudinal dispersion coefficient in a pipe reaches its asymptotic value at $T = 0.5$ and that the timescale is independent of the initial transverse distribution.

In this letter, the longitudinal dispersion coefficient is discussed in two aspects: the dimensionless history mean longitudinal transport coefficient (K) and the dimensionless temporal longitudinal transport coefficient (K_t). K is calculated as follows

$$K = \frac{\sigma^2(t)}{2t}\frac{D_z}{H^2U^2} \quad (4)$$

whereas K_t is calculated as

$$K_t = \frac{\mathrm{d}\sigma^2(t)}{2\mathrm{d}t}\frac{D_z}{H^2U^2} \quad (5)$$

The developments of K and K_t (i.e., the curve (Ad and D_z)) in Fig.2 show that the values of K and K_t increase to their asymptotic values at $T = 1.0$ and $T = 0.5$, respectively. The results by the Ad and