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Differential evolution approach to reliability-oriented optimal design

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ABSTRACT

The adoption of an evolutionary optimization approach, to tackle Reliability Based Design Optimization (RBDO) problems for which classical optimization is not feasible, can potentially lead to expand the use of RBDO in engineering applications. The herein proposed novel approach consists of coupling a differential evolution strategy with the one-level reliability method based on the Karush–Kuhn–Tucker (KKT) conditions. By selecting a few significant examples from the literature, convergence is found within a number of iterations compatible with the current capabilities of standard computational tools. These examples are chosen because lack of convergence is reported in literature when adopting classical gradient-based methods coupled with the reliability assessment. The performance and the efficiency of the algorithm are discussed, together with possible future improvements.

1. Introduction

Traditional design procedures of engineering systems are based on deterministic models and parameters. The variability of the loads, the material properties, the geometry, and the boundary conditions is included by means of modeling idealizations and simplifying assumptions, such as the consideration of mean or extreme values and the introduction of safety factors derived from past practice and experience. This approach is not able to realistically capture the influence of the uncertainties in the system parameters on the structural performance, which might result to be unsatisfactory. Furthermore, it cannot provide a quantitative measure of the system safety since it is unable to capture a mathematical relation with the risk assessments based on which decisional procedures are carried out [1].

Despite the development of a broad spectrum of different reliability-based design optimization (RBDO) methods in literature over the last 50 years (see, for example, [2]), their practical application to engineering problems is still rather limited in comparison with the deterministic design procedures. This is mainly due to the numerical challenges and computational cost often encountered when explicitly taking into account the effects of the unavoidable uncertainties in the structural performance. In Valdebenito and Schueller [3], the need to improve the current strategies for solving RBDO problems is stated, with particular emphasis on the aspects of numerical efficiency and robustness.

The selection of a particular optimization algorithm can be crucial to solve a specific RBDO problem. Indeed, the numerical efficiency, accuracy and stability of the solving algorithm determine the adequacy of the method to practical engineering applications. When traditional gradient-based algorithms are adopted for the objective function minimization, the efficiency related to the moment based methods comes at the price of a limited field of applications, which does not include problems characterized by a large number of random variables and multiple failure criteria considering nonlinear performance functions. Hence, there is a need to re-address the topic in the case that a different solution strategy, such as the one based on evolutionary algorithms of differential type, is pursued. The herein proposed novel approach consists of coupling a differential evolution (DE) strategy [4] with a one-level reliability method based on the Karush–Kuhn–Tucker (KKT) conditions [5].

The adoption of a one-level approach based on the direct introduction of the KKT conditions in the formulation of the general cost minimization problem was proposed in Kuschel and Rackwitz [6] for a single limit state function. Mathematical proof that the solution point of the FORM optimization problem fulfills the KKT conditions is given. The existence of an optimal design point guarantees that the vectors of the Lagrangian multipliers associated to the KKT conditions also exists. Nevertheless, the dependence of the failure domain on the design parameters prevents one from mathematically proving any general convergence property of the method. This observation holds also for the two-steps RBDO approaches [7], where two nested optimization problems are formulated, with the inner loop dedicated to the reliability analysis and the outer loop to the cost optimization.

Further adaptations were studied in order to handle problems with multiple independent failure modes and/or time-variant stationary loads [5]. The solution was sought by traditional gradient-based optimizers which required the computation of the gradient of the objective as well as the gradient of the constraints. In particular, taking the gradient of the second KKT optimality condition implicitly assumes the existence of the second order derivatives of the limit state functions. Such a requirement was identified as one of the

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main drawbacks of the one-level approach, thus suggesting that a heuristic search algorithm would be particularly suited to overcome this difficulty.

The application of the evolutionary strategies to RBDO problems has been studied mainly in association to a Monte Carlo simulation approach where the reliability calculations are carried out within a two-levels framework (see, for example, [8]). In the present work, the motivation of coupling a DE solution strategy to a one-level RBDO formulation is two-folds.

(1) By reformulating the optimization problem so that the convergence is simultaneously sought in both the random variables space and the design parameters space, the reliability analysis is avoided thus potentially reducing the overall computational cost.
(2) The DE approach can potentially broaden the reach of the RBDO method to those cases where multiple limit states and/or strong nonlinearities prevent the traditional gradient-based optimizer from reaching convergence.

Evidence for these two statements is sought by applying the proposed methodology to a few numerical examples for which the unfeasibility of classical optimization is openly stated in literature. For the selected numerical examples, convergence is found within a number of iterations compatible with the current capabilities of standard computational tools. Proving any general conclusion upon the convergence of the method is out of the scope of this work. Despite the insights derived from experimentation and the good convergence properties of the DE algorithm empirically observed by Storn and Price [4] from extensive testing under various conditions, a rigorous mathematical convergence proof still lacks and could be the object of future research. At present, heuristic tools, as the DE scheme adopted in this paper, come as solution providers without a sound mathematical foundation. For this reason their potential is often denied, or, alternatively, the methods are just used on a trial and error basis. Nevertheless, their capabilities in extending the reach of existing methods and permitting the feasibility of new approaches are of interest for many engineering fields that involve the solution of optimization problems.

The paper is structured as follows. In Section 2, the governing relations of the general RBDO problem are briefly provided. In Section 3, the strategy of using the DE technique to overcome the limitations of the one-level KKT approach is proposed. A formulation of the optimization problem suitable to this application is developed. In Section 4, the performance of the proposed solution technique is discussed with reference to numerical examples which are characterized by high nonlinearities due to both the analytical form of the limit state functions and the non-normality of the random variables.

2. Governing relations

Let $\mathbf{X}$ be the $1 \times N$ vector of arbitrarily distributed random variables and let $\mathbf{d}$ be the $1 \times D$ vector of the design variables, which can be either defined as independent deterministic variables or as parameters of the marginal probability distributions of the $\mathbf{X}$'s. As an example, one considers the failure modes of a portal frame [5]; from a probabilistic point of view, the loads and the plastic hinge moments are assumed as the random variables in vector $\mathbf{X}$, whereas the mean values of the beam and columns plastic moments are the design parameters in vector $\mathbf{d}$. The basic formulation of a typical RBDO problem takes the form:

$$\min_{\mathbf{d}} : \quad C(\mathbf{d})$$

$$\text{s.t.}: \quad \begin{cases} P_{fi} = \Pr[G_i(\mathbf{d},\mathbf{X}) \le 0] \le P_{fi}^T, & i = 1,\dots,m \\ h_j(\mathbf{d}) \le 0, & j = m+1,\dots,M \end{cases} \tag{1}$$

where $C(\mathbf{d})$ is the objective function (or cost function) which can represent either the initial cost of the structure upon construction or the expected value of its life-cycle cost, including the expected lifetime and the operating costs; $h_j(\mathbf{d})$ are the deterministic constraints and usually refer to the upper and lower bounds of the design variables; m is the number of limit states (or performance functions [8]), and M is the total number of constraints. In the above formulation, the key issue is represented by the computational effort required to evaluate the probabilistic constraints which restrict the feasible design region by bounding the probability, P_{fi}, of violating each i-th limit state, $G_i(\mathbf{d}, \mathbf{X})$, $i=1,\dots,m$, to not exceed the given admissible failure probability, P_{fi}^T. Indeed, in the time-invariant case, each failure probability, P_{fi}, $i=1,\dots,m$, is defined by the integral

$$P_{fi} = \Pr[G_i(\mathbf{d},\mathbf{X}) \le 0] = \int \cdots \int_{G_i(\mathbf{d},\mathbf{X}) \le 0} f_{\mathbf{X}}(\mathbf{x}|\mathbf{d}) d\mathbf{x} \tag{2}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function of the random variables $\mathbf{X}$ whose realizations are denoted with $\mathbf{x}$. It is worth noting that the joint probability density function may depend on $\mathbf{d}$ in the cases in which some of the parameters of the probability distributions (e.g., the mean values) are considered as design variables.

As mentioned above, in the two-level approach, the RBDO problem is tackled by formulating two nested optimization problems, where the inner loop is dedicated to the reliability analysis and the outer loop concerns the cost optimization. Since the exact computation of the integral in Eq. (2) is unpractical, approximate methods are applied and they can be either based on stochastic simulations or moment methods. The first-order reliability method (FORM) often provides an adequate accuracy and it is widely applied for the inner loop optimization. After transforming the random variables $\mathbf{X}$ into the uncorrelated and standardized normal variables, $\mathbf{U}=\mathbf{U}(\mathbf{X})$, whose realizations are denoted by $\mathbf{u}$, the reliability index is computed by solving a constrained optimization problem:

$$\min_{\mathbf{U}} : \quad ||\mathbf{U}||, \quad \text{s.t.}: \quad G_i(\mathbf{U}) = 0 \tag{3}$$

Typically, the HL-RF numerical method is adopted to carry out the FORM reliability analysis, because it requires no line search. The solution $\mathbf{u}_i^*$, $i=1, \dots, m$, represents the most probable failure point for the considered i-th limit state, and the reliability index β_i is given by $\| \mathbf{u}_i^* \|$. According to the FORM approximation, the failure probability can then be estimated by $P_{fi} \sim \Phi(-\beta_i)$, where $\Phi(.)$ is the standard Gaussian distribution. Alternatively, the probabilistic constraint in Eq. (1) can be reformulated in terms of the reliability index, which must not be less than a threshold value, β_i^T, given by $\Phi^{-1}(1-P_{f\,i}^T)$. Then, the discussed two-loops method takes the name of the Reliability Index Approach (RIA) as reported in literature (see, for example, [9]).

The adoption of a two-levels strategy implies the repeated evaluation of the performance functions based on the mechanical model of the structure. Therefore, it becomes unpractical when finite element models of real structures with material and/or geometrical nonlinearities are considered. The introduction of an explicit model, such as the response surface, to approximate the implicit model representative of the outcomes of the finite element analyses can largely accelerate the inner reliability loop [7], but the computational cost is still high due to the evaluation of the response surface terms and the introduction of approximation errors. Recent studies targeted to the development of efficient reliability estimation approaches to be used in the framework of