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# Approaches to rare events in stochastic dynamics of ships

Vadim Belenky[a,*], Kenneth M. Weems[b], Christopher C. Bassler[a], Martin J. Dipper[a], Bradley L. Campbell[a], Kostas J. Spyrou[c]

[a] David Taylor Model Basin - NSWCCD, 9500 Macarthur Blvd., W. Bethesda, MD, USA
[b] Science Application International Corporation, 4321 Collington Rd, Bowie, MD, USA
[c] National Technical University of Athens, 9 Iroon Polytechneiou, Zographos, 15780, Greece

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ABSTRACT

The paper reviews problems and solutions related to extreme ship motions in random waves. In its general form, the dynamical system is described by integro-differential equations. The bandwidth of excitation is medium; stiffness is extremely nonlinear and random. In standard ship design, the main tools for the engineering evaluation are model experiments and numerical simulations using potential flow hydrodynamics with empirical models for non-potential flow forces. However, a direct Monte Carlo approach is impractical because of the high cost of running these tools and the rarity of extreme motion events. To obtain a practical solution, the principle of separation can be used to effectively consider the nonlinear phenomena resulting in an extreme response and the conditions that lead to the occurrence of such phenomena. This paper discusses fundamental aspects of three methods that use the principle of separation: the peaks-over-threshold/envelope peaks-over-threshold method, the split-time method, and the critical wave group method.

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## 1. Introduction

The objective of this paper is to present an overview of the state-of-the art in the field of extreme motions of ships in random waves. The results reviewed in the paper have been already reported elsewhere and the intention is to "integrate" them into a relatively concise text that can be presented to the wider scientific community.

### 1.1. Brief historical background

The motion of a ship in waves involves a number of hydrodynamic forces of different nature and represents a complex physical phenomenon. The study of ship motions has been pursued in mechanics since the mid-1700's, starting with the works of Bouguer [1], Euler [2], and Bernoulli [3]. However, it was not until the second half of the 19th century that the theory of ship motions was created by the work of Froude [4] and Krylov [5]. This ship motion theory considered a sinusoidal wave of small amplitude and assumed that the presence of a ship does not affect the pressure in the incident wave. This theory was later extended to account for the wave–ship interaction in a form of the ship's wave diffraction and the wave radiation due to the ship's motion [6–8]. The extension of this theory to irregular waves was developed by St. Denis and Pierson [9], completing the linear theory of ship motions in the frequency domain.

Subsequent work has expanded into the time domain. As in many other areas of applied mechanics, the linear theory of ship motions uses the assumption that the amplitude of the ship motions is small. Further development toward large amplitude motions has brought the problem into the framework of numerical hydrodynamics in the time domain [10]. This has made the Monte Carlo method with potential flow hydrodynamics the principal tool for the evaluation of large ship motions in irregular waves.

### 1.2. Forces acting on a ship in waves

Periodic ship motions are excited by the pulsating pressure of the wave field around the ship. The integration of the wave pressure over the instantaneous submerged portion of ship hull surface gives the principal vector and moment of the excitation force.

Traditionally, this force is considered as a sum of two parts. The first part, called the "Froude–Krylov" (F–K) force, assumes that a ship does not disturb the wave pressure field by her presence, and is calculated by integrating the undisturbed incident wave pressure over the ship's hull. As the size of a ship may be comparable with the length of the wave, the wave will be diffracted by the ship as it would be by any other obstacle. In addition, the motion of the ship, like any other body moving on the surface of fluid, will generate waves that radiate away. These diffraction and radiation waves interfere with the incoming wave

* Corresponding author. Tel.: +1 301 227 1720; fax: +1 301 227 5442.
*E-mail address:* vadim.belenky@navy.mil (V. Belenky).

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and change the pressure field surrounding the ship. As a result, the real excitation is different from the F–K force by the second part, called the "hydrodynamic", or "wave-body disturbance" forces, which account for the ship's influence on the wave.

The radiation forces for a ship include a component which is proportional to and opposes the acceleration of the body. Since this force, which is known as "added mass", is often of comparable magnitude to the inertial forces, it is often included with the inertial forces by introducing an added mass term that is dependent on the frequency of the motions.

A ship without forward speed in calm water will float at static equilibrium; her weight is counteracted by her buoyancy force. A wave excites motions and disturbs her equilibrium. As a result, principal vector and moment of the buoyancy force creates a stiffness term in the dynamical system. Because the buoyancy force is an integral of the hydrostatic pressure over the instantaneous submerged portion of the ship's hull, the separation of the stiffness part and F–K part of the excitation is not possible, strictly speaking, beyond linear formulation. Hydrostatic stiffness only exists for the three "vertical" degrees of freedom: heave, roll, and pitch.

Damping of ship motions is caused by three different physical phenomena: energy loss from wave radiation, vortex generation from lifting appendages and flow separation, and viscous friction on the hull surface. As the instantaneous submerged portion of the hull changes so does the damping, as it depends on the derivatives of the motion displacements.

A ship performing a turn or other maneuver in the horizontal plane will be affected by vortices shed from the submerged portion of the hull and appendages, such as bilge keels and skegs. These maneuvering forces depend on the derivatives of motions, as well as the draft and roll of the ship. Other maneuvering forces acting on the ship include thrust (from propeller, water jet, or azipod) and the steering force generated by a rudder, nozzle, bow thruster, or other device. Aerodynamic forces from wind, which also have vortical origins, can have a significant effect in roll motions in severe storm conditions.

### 1.3. Dynamics of a ship in waves

The complex nature of the forces acting on a ship in waves begets complex dynamics. Since ship behavior in waves is described with integro-differential equations that are too difficult to study directly, these dynamics are usually investigated with ordinary differential equations, representing a certain level of approximation.

Roll stiffness usually has two or more stable equilibria. The existence of two or more stable equilibria invokes the possibility of transitions between them. For most practical cases, one of these stable equilibria is the ship's capsized position. Thus total stability failure is, in fact, associated with the transition to another stable equilibrium in roll. Pitch stiffness also always includes multiple stable equilibria for ship-like structures; a similar mechanism in pitch is responsible for pitchpoling, the nautical term for flipping over forward.

The existence of stable and unstable equilibria in roll makes roll stiffness strongly nonlinear. Both softening and hardening springs can be encountered, depending on the hull geometry; the strongest effect is from freeboard height. The fundamental dynamics resemble those of a nonlinear rotational oscillator that, as well known, away from the neighborhood of stable equilibrium can give rise to fold and flip bifurcations. Flip bifurcation may lead to deterministic chaos. Furthermore, the invariant manifolds of the unstable equilibria may, under the effect of some external forcing, intersect, thereby causing the geometry of the attraction domain to go fractal and diminish [11–13].

When a ship is sailing in stern or bow quartering seas, roll stiffness changes significantly with the ship's motion in the waves. This is one of the possible mechanisms to create parametric excitation, leading to parametric resonance in roll. The phenomenon of parametric roll was known to Naval Architects since at least the 1930's [14–16], with the first fundamental studies appearing in the 1950's [17,18]. The next milestones were experimental observation [19] and probabilistic treatment [20,21]. Interest in this phenomenon has significantly increased after an accident with a large container ship in 1998 [22], focusing further on applied aspects [23–25]. A brief review on the probabilistic treatment of parametric roll up to about 2005 can be found in [25].

The decrease of roll stiffness can be large enough to make the dynamical system very soft. If a ship is sailing in stern quartering seas, the encounter frequency may be low due to the Doppler effect; this situation may lead to prolonged exposure to significantly decreased roll stiffness, creating a very large roll angle or even capsizing the ship [26].

When a ship is sailing straight in calm water with constant speed, the thrust of her propulsion is counteracted by water resistance (composed mostly from wave, vortex, and frictional components) in dynamic equilibrium. When a ship is sailing in waves, she experiences oscillatory surge motions about this dynamic equilibrium, though the equilibrium may be altered in bow quartering seas by the additional resistance caused by waves.

When a ship is sailing in stern quartering overtaking waves, the F–K force in surge may create a dynamical equilibrium for the speed equal to wave celerity. If a ship is attracted to this equilibrium, she rides with the wave realizing the phenomenon known as surf-riding. In many cases, the ship is directionally unstable in this situation, as the dynamic equilibrium repels her in yaw. This can cause the ship to experience an uncontrollable sharp turn known as a broaching-to. During such a turn, centrifugal forces may cause the ship to develop a large roll angle or even capsize. A historical perspective on broaching-to from the time of tall ships can be found in [27]. While research on broaching-to was regularly carried out since the 1940's [28,29], complete understanding and full description of the phenomenon from nonlinear dynamics perspective became available only in mid-1990's [30,31]. It was also explained how broaching-to can occur without surf-riding (something that had been empirically observed from real incidents), caused by a fold bifurcation and jump in the amplitude of yaw motion [32].

## 2. The problem of rarity

The main difficulty with the assessment of dynamically-related undesirable events like the above, or dynamic "failures", is both their rarity and significant nonlinearity, which need to be addressed simultaneously.

### 2.1. Nonlinearities and the problem of rarity

Failures related to a ship's motions and loads in severe, irregular seas are characterized by both their rarity of occurrence and their significant nonlinearity. Because of this, the accurate evaluation of the ship response in these conditions is difficult and it becomes impractical to use traditional "brute-force" direct assessment methods such as Monte Carlo simulations and/or a very large number of experimental realizations. Assessing the dynamical response to these wave sequences constitutes the general problem of rarity—when the time between events is long, compared to a relative time-scale [33].

Nonlinearity makes it difficult to use traditional techniques to determine values associated with rare events, such as extreme value distributions. While the theory of extreme distributions is still applicable, the fitting of these distributions may be difficult