Probabilistic Engineering Mechanics 28 (2012) 164–168

Contents lists available at SciVerse ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

Reliability analysis of large structural systems

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ARTICLE INFO

Article history:
Received 10 June 2011
Received in revised form
14 July 2011
Accepted 17 August 2011
Available online 7 September 2011

Keywords:
System reliability
Large scale system
Monte Carlo simulation
Failure probability
Statistical estimation

ABSTRACT

Brute force Monte Carlo simulation methods can, in principle, be used to calculate accurately the reliability of complicated structural systems, but the computational burden may be prohibitive. A new Monte Carlo based method for estimating system reliability that aims at reducing the computational cost is therefore proposed. It exploits the regularity of tail probabilities to set up an approximation procedure for the prediction of the far tail failure probabilities based on the estimates of the failure probabilities obtained by Monte Carlo simulation at more moderate levels. In this paper, the usefulness and accuracy of the estimation method is illustrated by application to a particular example of a structure with several thousand potentially critical limit state functions. The effect of varying the correlation of the load components is also investigated.

1. Introduction

In general, it is very difficult to calculate the reliability of realistic structural systems by using conventional theoretical reliability techniques. This is usually caused by the high number of basic random variables needed for modeling the uncertain elements of the problem and the large number of safety margins that are used to describe the system, which will generally consist of a series system of parallel subsystems. At least, in principle, the reliability of complicated structural systems can be accurately predicted in a straightforward manner by standard Monte Carlo simulation methods. However, the computational burden may be prohibitive for highly reliable systems. Motivated by this observation, the authors have initiated the development of simulation based methods for calculating the reliability of structural systems that aims at reducing the computational cost. This is achieved by introducing a cascade of systems depending on a parameter varying between zero and one, where the original system is obtained when this parameter equals one. When the parameter value is zero, the system is highly prone to failure, and for the small to intermediate values of the parameter, the failure probability can be estimated with high accuracy by Monte Carlo simulation with moderate computational efforts. By exploiting the regularity of the failure probability as a function of the parameter, an approximation procedure for predicting the failure probability at parameter value one has been proposed in [1]. This method was applied to small structural systems in [1,2] and it was shown that good results could be quickly obtained on a standard laptop computer.

In this paper, the specific case of a 3D beam structure (grillage) will be studied. It consists of 40 transverse and 40 longitudinal beams, creating 40×40 equidistant and intersecting main beams. As discussed in the section on numerical examples, the number of basic random variables in the model will be 4880 and the number of limit state functions will be 6540. At the present stage of development, this system would represent a huge challenge for any standard reliability method for system analysis. It will be shown that the proposed method can handle also this system with feasible computational efforts.

2. Component reliability

The basic element in the construction of the cascade of systems mentioned in the Introduction is already seen for the case of component reliability. Let us consider a safety margin M which represents the failure mode under consideration. It is assumed that M is expressed in terms of n basic random variables $X_1, \ldots, X_n$ as $M = G(X_1, \ldots, X_n)$, where the limit state function G can be rather complicated. To calculate the failure probability $p_f = \text{Prob}(M \leq 0)$, the method proposed in [1] is based on introducing a cascade of safety margins $M(\lambda) = M - \mu_M(1 - \lambda)$, where $\mu_M = \text{E}[M]$

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doi:10.1016/j.probengmech.2011.08.024

and the parameter $\lambda \in [0, 1]$. Putting $p_f(\lambda) = \text{Prob}(M(\lambda) \leq 0)$, then $p_f = p_f(1)$. It is seen that $\text{E}[M(0)] = 0$, which means that the corresponding component is highly prone to failure when $\lambda = 0$. Hence, the failure probability $p_f(\lambda)$ will decrease from a high value at $\lambda = 0$ to the small target value at $\lambda = 1$. As proposed in [1], we shall now make the following assumption about the behavior of the failure probability $p_f(\lambda)$ of the safety margin $M(\lambda)$:

$$p_f(\lambda) \underset{\lambda \to 1}{\approx} q(\lambda) \exp\left\{-a(\lambda - b)^c\right\}, \tag{1}$$

where the function $q(\lambda)$ is slowly varying compared with the exponential function $\exp\{-a(\lambda - b)^c\}$.

The practical importance of this relation, if it applies, is that the target failure probability $p_f = p_f(1)$ can be obtained from values of $p_f(\lambda)$ for $\lambda < 1$. Our focus, in this paper, is on methods for estimating p_f by Monte Carlo simulation. The observation above may then be significant, as it is usually easier to estimate $p_f(\lambda)$ for $\lambda < 1$ more accurately than the target value since they are larger. Fitting the parametric form for $p_f(\lambda)$ to the estimated values would then allow us to provide an estimate of the target value by extrapolation. The viability of this proposal will be demonstrated by a numerical example below. However, there could be exceptional cases, e.g., for variables which are uniformly distributed on a finite interval, where application of a functional form like Eq. (1) might not be accurate for all probability levels. It is believed that such discrepancies can be identified at an early stage of the probabilistic analysis.

3. System reliability

We shall consider structural systems that can be described as combinations of component failure modes. Each failure mode is assumed to be represented by a safety margin. Let $M_j = G_j(X_1, \ldots, X_n)$, $j = 1, \ldots, m$, be a set of m given safety margins expressed in terms of the n basic random variables that are involved in specifying the structural system we consider. The cascade of systems we shall use in our method for estimating the failure probability is then obtained from the class of safety margins defined as $M_j(\lambda) = M_j - \mu_j(1 - \lambda)$, where $\mu_j = \text{E}[M_j]$.

The basic systems are series systems and parallel systems. If the system at hand is a series system, the failure probability will be given as,

$$p_f(\lambda) = \text{Prob}\left(\bigcup_{j=1}^{m} \{M_j(\lambda) \leq 0\}\right). \tag{2}$$

In general, any system can be written as a series system of parallel subsystems [3–5]. The failure probability would then be given as,

$$p_f(\lambda) = \text{Prob}\left(\bigcup_{j=1}^{l} \bigcap_{i \in C_j} \{M_i(\lambda) \leq 0\}\right) \tag{3}$$

where $C_j \subseteq \{1, \ldots, m\}$, $j = 1, \ldots, l$, denote the index sets defining the parallel subsystems.

To use the proposed method for failure probability estimation, we then make the assumption that $p_f(\lambda)$ can be represented as in Eq. (1) also for the system reliability problem.

4. Reliability estimation by optimized fitting

The method to be described in this section is based on the assumption expressed by Eq. (1). As argued in [1], for practical applications it is implemented in the following form,

$$p_f(\lambda) \approx q \exp\left\{-a(\lambda - b)^c\right\}, \tag{4}$$

for $\lambda_0 \leq \lambda \leq 1$ for a suitable value of λ_0, where q is now assumed to be a constant. An important part of the method is therefore to identify a suitable λ_0 so that the rhs of Eq. (4) represents a good approximation of $p_f(\lambda)$ for $\lambda \in [\lambda_0, 1]$.

For a sample of size N of the vector of basic random variables $\mathbf{X} = (X_1, \ldots, X_n)$, let $N_f(\lambda)$ denote the number of samples in the failure domain of $M(\lambda)$. The estimate of the failure probability is then

$$\hat{p}_f(\lambda) = \frac{N_f(\lambda)}{N}. \tag{5}$$

The coefficient of variation of this estimator is

$$C_V(\hat{p}_f(\lambda)) = \sqrt{\frac{1 - p_f(\lambda)}{p_f(\lambda) N}}. \tag{6}$$

A fair approximation of the 95% confidence interval for the value $p_f(\lambda)$ can be obtained as $\text{CI}_{0.95}(\lambda) = \left(C^-(\lambda), C^+(\lambda)\right)$, where

$$C^{\pm}(\lambda) = \hat{p}_f(\lambda)\left(1 \pm 1.96 C_V(\hat{p}_f(\lambda))\right). \tag{7}$$

The problem of finding the optimal values of the parameters q, a, b, c can carried out by optimizing the fit on the log level by minimizing the following mean square error function [1],

$$F(q, a, b, c) = \sum_{j=1}^{M} w_j \left(\log \hat{p}_f(\lambda_j) - \log q + a(\lambda_j - b)^c\right)^2, \tag{8}$$

where $\lambda_0 \leq \lambda_1 < \cdots < \lambda_M < 1$ denotes the set of λ values where the failure probability is empirically estimated. w_j, $j = 1, \ldots, M$, denote weight factors that put more emphasis on the more reliable data points, alleviating the heteroscedasticity of the estimation problem at hand. The choice of weight factors is to some extent arbitrary. In this paper, we use $w_j = \left(\log C^+(\lambda_j) - \log C^-(\lambda_j)\right)^{-\theta}$ with $\theta = 1$ and 2, combined with a Levenberg–Marquardt least squares optimization method [6]. This usually works well provided reasonable, initial values for the parameters are chosen. In this paper, $\theta = 2$ has been used. Note that the form of w_j puts some restriction on the use of the data. Usually, there is a level λ_j beyond which w_j is no longer defined. Hence, the summation in Eq. (8) has to stop before that happens. Also, the data should be preconditioned by establishing the tail marker λ_0 in a sensible way.

Although the Levenberg–Marquardt method, as described above, generally works well, it may be simplified by exploiting the structure of F. It is realized by scrutinizing Eq. (8) that if b and c are fixed, the optimization problem reduces to a standard weighted linear regression problem. That is, with both b and c fixed, the optimal values of a and $\log q$ are found using closed form weighted linear regression formulas in terms of w_j, $y_j = \log \hat{p}_f(\lambda_j)$ and $x_j = (\lambda_j - b)^c$.

It is obtained that the optimal values of a and q are given by the relations,

$$a^*(b, c) = -\frac{\sum_{j=1}^{M} w_j (x_j - \bar{x})(y_j - \bar{y})}{\sum_{j=1}^{M} w_j (x_j - \bar{x})^2}, \tag{9}$$

and

$$\log q^*(b, c) = \bar{y} + a^*(b, c)\bar{x}, \tag{10}$$

where $\bar{x} = \sum_{j=1}^{M} w_j x_j / \sum_{j=1}^{M} w_j$, $\bar{y} = \sum_{j=1}^{M} w_j y_j / \sum_{j=1}^{M} w_j$.

The Levenberg–Marquardt method may now be used on the function $\tilde{F}(b, c) = F(q^*(b, c), a^*(b, c), b, c)$ to find the optimal values b^* and c^*, and then the corresponding a^* and q^* can be calculated from Eqs. (9) and (10).

For estimation of the confidence interval for a predicted value of the failure probability provided by the optimal curve, the empirical confidence band is reanchored to the optimal curve. The range of fitted curves that stay within the reanchored confidence band will determine an optimized confidence interval of the predicted value. This is obtained by constrained nonlinear optimization. As a final