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Estimation of repairable system availability within fixed time horizon

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Abstract

The paper presents an efficient approach to estimating the availability of a repairable system observed within a fixed time horizon, which is regarded as a random variable over range [0, 1]. A Beta distribution based approximation of this characteristic is proposed. The presented results are derived from Monte Carlo simulation and statistical analysis of the simulation results. Numerical examples are also provided to illustrate the practical impact of this approach.

Keywords: Fixed time horizon availability; System uptime; Service level agreement

1. Introduction

Judging from the practical perspective, availability is one of the most interesting characteristics of a repairable system. Although the steady-state availability of a simple model of alternating renewal process is given as a standard result in reliability textbooks (e.g. [1,2]), some authors develop related measures to address other problem formulations, e.g. mission availability, random-request availability [3], or alternatively, to discuss the system availability for relaxed assumptions, e.g. for nonexponential outage distribution [4,5]. Recently, many authors consider the problem of system structure optimization or maintenance strategy optimization to meet desired availability constrains under minimal cost [6–8].

In this paper availability of a repairable system is considered in a fixed time horizon. Our aim is to develop a practical and easy to use method to obtain estimates for a real-life system of

- 'guaranteed' availability that it is expected to exhibit during a specified time interval, or, alternatively,
- risk that a system will perform below a defined availability threshold in a specified time interval.

Related problem was considered by Takacs [9] and Muth [10,11]. They considered distribution of the total downtime or uptime in a specified time interval and showed that this distribution approaches the Gaussian distribution for long time horizons. In case of shorter time horizons the theoretical results require the calculation of convolutions of uptime and downtime distributions and as such are numerically complicated. An approximated formula for uptime distribution was given by Funaki [12]. An approximation obtained by modelling the renewal process with a compound Bernoulli process was developed by Jeske [13].

In the presented paper the distribution of fixed horizon system availability is approximated using Beta distribution. Useful estimators of Beta distribution parameters are proposed, based on the results of Monte Carlo simulation.

Results obtained here can be applied to:

- Analysis of time redundant systems—for instance, to estimate the risk that a repairable system fails to complete a task in constrained time due to excessive downtime. Time redundancy is receiving considerable attention in reliability literature [11,14].
- Estimation of risk of not conforming to system availability commitments specified in service level agreements (SLA). SLA commonly use system availability as the basic measure of service quality.

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Nomenclature

(t_0, t_0+T)	time horizon is which system is observed
$A, \overline{A}$	availability of system in the fixed time horizon (a random variable that takes values from the range $[0, 1]$) and its mean value
$F(x), G(x)$	the distributions of the time to system failure and the repair time
λ, μ	failure and repair rate of system
$a(t), a$	availability function of an alternating renewal process and its steady-state value

2. Problem formulation

Let us consider the behaviour of a repairable system within a fixed time horizon, i.e. beginning from time t_0 until time $t_0 + T$. Within this time period the system may fail a random number of times N and be subsequently repaired. The cumulative uptime τ_{up} is the sum of all the periods when the system is running within the considered time horizon.

System availability over a fixed time horizon is defined as the ratio of the cumulative uptime to the length of the fixed time period:

$$A = \frac{\tau_{\mathrm{up}}}{T}.$$

It should be noted that the fixed horizon availability is a random variable (as a ratio of a variate and a fixed value), unlike the availability function normally used to analyse the repairable systems. Yet, there is a simple relationship between the mean value $\overline{A}$ of fixed horizon availability and the availability function:

$$\overline{A} = \frac{1}{T}\int_{t_0}^{t_0+T} a(t)\,\mathrm{d}t. \tag{1}$$

The distribution of variable A has some other fairly obvious properties. Since the uptime cannot be negative or exceed the considered time horizon, so the variable may assume values in the range of $[0, 1]$. Furthermore, the probability that the value is exactly 1 corresponds to the situation that there is no failure in the considered time horizon, i.e.

$$\Pr\{A = 1\} = a(t_0)(1 - F(T)), \tag{2}$$

and the probability that the value is exactly 0 corresponds to system being initially failed and not being repaired in the considered horizon, i.e.

$$\Pr\{A = 0\} = (1 - a(t_0))(1 - G(T)), \tag{3}$$

assuming that time to failure and repair time are independent random variables. Naturally, in normal cases the failure state probability is very small and hence we may assume that $\Pr\{A = 0\} \approx 0$.

It should be noted that it follows from Eqs. (2) and (3) that A does not have a continuous distribution—it has discontinuities at the borders. In fact, the distribution of the fixed period availability is fairly complex: it is limited to the range $[0, 1]$, it has discrete probabilities at the end points and has a continuous distribution dependent on $F(T)$ and $G(T)$ within the open range $(0, 1)$.

2.1. Stationary alternating renewal process

The general formulation of the problem, as presented above, was insufficient for conducting numerical experiments. To this end it was necessary to assume specific properties of the distributions of failure and repair times. The presented experiments were conducted on a stationary alternating renewal process, i.e. a system with independent, exponential distributions of failure and repair times. In this case Eqs. (1)–(3) take the form of

$$\overline{A} = a = \frac{\mu}{\lambda + \mu}, \tag{4}$$

$$\Pr\{A = 1\} = \frac{\mu}{\lambda + \mu}\exp(-\lambda T), \tag{5}$$

$$\Pr\{A = 0\} = \frac{\lambda}{\lambda + \mu}\exp(-\mu T). \tag{6}$$

As already noted, it may be assumed in all the practical cases that $\Pr\{A = 0\} \approx 0$. For this reason the considerations are limited to two specific cases:

- *Case* I (*when the probability* $\Pr\{A = 1\} \approx 0$): This occurs if $\exp(-\lambda T) \approx 0$, that is when the time horizon T is sufficiently long in comparison to the mean time to failure. This assumption is very significant, as it eliminates the discontinuities at the ends of the $[0, 1]$ range.
  In this case the considered distribution may be justifiably approximated with a class of fixed range continuous distribution functions. Such approximation is very desirable when it is necessary to make management decisions based on availability analysis, as it is further shown. In Section 3, the Beta distribution is used for this approximation. The choice was purely empirical—it resulted in best fit as compared to other analytically defined fixed range distributions.
- *Case* II (*when* $\Pr\{A = 1\} > 0$): This occurs when the considered time horizon is of the same order of magnitude as the mean time to failure. The Beta function, which is continuous, cannot be fitted to approximate such a distribution directly.

3. Estimation of system availability distribution

Sample histograms of system availability obtained by Monte Carlo simulation are shown in Fig. 1, with kernel density estimators fitted for the purpose of illustration.