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Time domain series system definition and gear set reliability modeling

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ABSTRACT

Time-dependent multi-configuration is a typical feature for mechanical systems such as gear trains and chain drives. As a series system, a gear train is distinct from a traditional series system, such as a chain, in load transmission path, system-component relationship, system functioning manner, as well as time-dependent system configuration. Firstly, the present paper defines time-domain series system to which the traditional series system reliability model is not adequate. Then, system specific reliability modeling technique is proposed for gear sets, including component (tooth) and subsystem (tooth-pair) load history description, material priori/posterior strength expression, time-dependent and system specific load-strength interference analysis, as well as statistically dependent failure events treatment. Consequently, several system reliability models are developed for gear sets with different tooth numbers in the scenario of tooth root material ultimate tensile strength failure. The application of the models is discussed in the last part, and the differences between the system specific reliability model and the traditional series system reliability model are illustrated by virtue of several numerical examples.

1. Introduction

Multi-state is an important system feature for reliability evaluation, on which lots of investigations have been focused, including reliability modeling, reliability estimation and reliability optimization etc [1–3]. Multi-configuration is another important system feature that has not yet been sufficiently emphasized. Investigations on multi-configuration system reliability modeling are very few, though the so called "active system" [4] such as gear trains has long been recognized as a special kind of system in terms of system function.

Traditional system reliability models are mostly developed in the scenario of electronic equipment, control systems, as well as execution systems composed of active components such as pumps, valves and relays. A common feature of these systems is the constant system-component configuration and changeless system-component functioning relationship.

Mechanical systems or "active systems" are different from structural systems or electronic systems in the time dependency of system-component relationship. Machinery and mechanical mechanisms usually involve some kind of motion and present varying configuration, varying loading path, and varying system-component connection during operation. A gear train is a typical type of mechanical system or "active system". A gear train, a gear set, or even a gear should be taken as a series system since any tooth failure will cause system failure. However, it is different from a traditional series system such as a chain, because a gear transmission system (e.g. a gear set) is composed of different components (meshing tooth-pairs) in different time intervals, and all the components do not operate (bear load and transmit movement) simultaneously. In contrast, a traditional series system, such as a chain, is composed of the same components always, and all the components are subject to the same load as long as the system is in operation. For a multistage gear transmission system such as a gear box applied in a car or a truck, different tooth-pairs of different gear-sets engage in different time intervals, i.e. the system is composed of different tooth-pairs of different gear-sets in different time intervals. Such systems are defined as time-domain system in this paper, and the typical one is time-domain series system, such as a gear set.

In the simplest situation that a gear (a system with m teeth) subjects to one time of load action and only one tooth (component) bears the load, the reliability of a gear of m teeth (system) equals to that of a tooth (component). Differing from this, the reliability of a chain (system) of m links (components) does not equal to that of a link even in the situation of one time of load action, since all the links are subject to the load and at risk of failure. By virtue of this example, it is easy to understand that traditional series system reliability model is not appropriate for gear transmission systems.

For a gear set in transmission, the different tooth-pairs, which

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constitute two-component series subsystem, mesh by turns. This kind of series system is composed of different subsystems (tooth-pairs) in different time intervals, and only two components (one subsystem, i.e. one pair of meshing teeth) operate in a given time interval provided that the gear contact ratio equals to one. As to a gear in a gear set of which contact ratio equals to one, it is a “component” at any time point or a certain small time interval since only the meshing tooth is in work state and subjects to load, but it is a series system in a longer time period since any tooth failure means the transmission function failure of the gear.

Another issue involved in system reliability modeling is the statistical dependence among component failures. The majority of classical system reliability models were developed with the assumption of independent component failures. However, the so called common cause failure or common mode failure effect are normally ignorable for system reliability modeling [5–7].

As to the reliability evaluation of mechanical transmission systems, the same reliability model as that applied to electronic systems has been widely employed [8–10], without realizing the particular, periodically varying system-component relationship in mechanical transmission systems. For instance, stress-strength interference analysis method was applied to calculate gear transmission system reliability, implying that a gear is equivalent to a component [11–14] and only one tooth is considered for gear reliability estimation [15–17]. In fact, simplification has long been the common practice for complex system reliability analysis, though variety of errors may be brought about. Recently, more and more system or failure mode/mechanism specific reliability analysis techniques are presented, such as evaluating the reliability of an RF semiconductor device when several different mechanisms contribute simultaneously to its wearout [18], evaluating the reliability of multi-body mechanisms [19], and identifying appropriate reliability model for massive multiple repairable units [20].

As traditional system reliability modeling has not yet specially concerned on gear transmission system, the present paper develops reliability models for this kind of time-domain series systems. The failure mode considered is tooth root material fracture.

2. Reliability modeling oriented system feature

For reliability assessment, the basic system feature is characterized by the relationship between system functional state and component functional state. The relationship between system and its components might be very simple as that of a chain composed of a number of identical links, or quite complex as that of some mechanical equipment. Systems are conventionally classified as serial system, parallel system, k-out-of-n redundant system, etc. based on the relationship between system functional state (operation or failure) and component functional state. Usually, the configuration of a system concerned in reliability analysis, i.e. the system's functional structure constructed by its components, as well as the connection between system and components, is constant. In other words, neither the system configuration nor the system-component connection varies during operation. A typical traditional series system is a chain composed of a number of links, all of the components (links) are exposed to the same load when the system (chain) is in operation, and all components are at risk of failure at any time during system (the chain) operation. To this kind of series systems, i.e. the traditional series system having changeless configuration and changeless loading path, reliability model has been well developed.

In contrast to a chain, a gear set is a different kind of series system in terms of reliability modeling. In a gear set (a gear transmission system composed of two gears), all the components (the teeth on the two gears) do not work at the same time. Instead, different tooth-pairs play their role (mesh) by turns. For one gear, it is a component at any moment since only one tooth is in operation state (provided that the contact factor equals to one for the gear set); while it is a series system in its operation process (a rather long time interval) since all the teeth have to function and the failure of any tooth will lead to system (gear) failure. A system with such feature is defined as a time-domain series system in the present paper.

If the load subjected to a gear set is constant, i.e. all the teeth on a gear subject to the same load when meshing with the teeth on the other gear, a time-domain series system can be taken as a traditional series system for reliability estimation, at least for a gear set of which the two gears have the same number of teeth. If load is random, as the tooth-pairs get into operation by turns, the loads subjected to the individual meshing tooth-pairs are different, the reliability model of a time-domain series system may considerably differ from that of a traditional series system. Besides, different tooth numbers of the two gears will make complex meshing teeth combinations for a gear set, and lead to different load action numbers to the every teeth of the different gears, making the reliability model more complicated.

3. Preliminary to gear transmission system reliability modeling

3.1. Gear tooth reliability

During gear transmission, the load subjected to a gear can be described by a variable amplitude load history, and the load subjected to a meshing tooth-pair can be taken as a random variable. Let X denote the material strength of tooth root, $f(x)$ denote its probability density function (pdf), Y denote the stress at the tooth root, and $g(y)$ denote its pdf. The reliability of a gear tooth associated with one time of load action, i.e. the probability of the strength being greater than the stress can be calculated by the traditional stress-strength interference model

$$R_t(1) = \int_0^\infty g(y) \int_y^\infty f(x)dxdy \tag{1}$$

For n times of load action under a specified variable amplitude load history, gear tooth reliability equals to the probability that all the n stress values are less than the strength, or the probability that the maximum extreme statistic of the n stress samples is less than the strength, i.e.

$$R_t(n) = \int_0^\infty g^{(n)}(y) \int_y^\infty f(x)dxdy \tag{2}$$

where $g^{(n)}(y) = n[G(y)]^{n-1}g(y)$ stands for the pdf of the maximum extreme statistic of n stress samples, $G(y)$ the cumulative distribution function (cdf) of the stress random variable.

3.2. Gear reliability

For a gear with m teeth, the strengths of the teeth are independent and identically distributed (iid) random variables. In gear transmission process, the functioning of a gear requires the functioning of all its teeth, thus all the teeth of the gear constitute a series system. Different from a traditional series system, the load action number to a component (a tooth) is different from that to the system (the gear). For the total n times of load action to a gear, the number of load actions to each of the tooth is n/m (exactly, an integer $int\{n/m\}$). During gear operation under a specified variable amplitude load history, tooth failures are independent events. Therefore, the reliability of a gear with m teeth corresponding to n times of load action equals to the product of the probabilities of