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Deriving the absolute wave spectrum from an encountered distribution of wave energy spectral densities

Ulrik D. Nielsen[a,b,*]

[a] DTU Mechanical Engineering, Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark
[b] Centre for Autonomous Marine Operations, NTNU AMOS, NO-7491, Trondheim, Norway

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ABSTRACT

The objective of ship motion-based wave spectrum estimation is to provide the distribution of wave energy densities in *absolute domain*. However, as a ship generally advances relative to the progressing waves, any spectrum estimate inherently dates back to the *encounter domain* and, consequently, the spectrum estimate must be transformed to absolute domain. In following sea conditions, spectrum transformation from encounter to absolute domain has no unique (mathematical) solution. This article presents an optimisation-based technique to carry out the particular transformation in following sea conditions. The optimisation relies on an object function established using (wave) spectral moments; calculated directly using the estimated encounter-wave spectrum on the one side and by using a parameterised wave spectrum valid in absolute domain on the other side. The simplicity of the transformation technique is a strength in itself as it leads to an insignificant computational effort in the transformation to absolute domain. Equally important, the specific technique proves capable to provide accurate results in the majority of cases, when comprehensive testings with numerically simulated data of following sea conditions are performed. Furthermore, the technique is tested successfully using experimental full-scale sea trials data.

1. Introduction

When a ship advances in its seaway (on deep water), the relative velocity and the angle between vessel and the progressing waves are the parameters that determine the observed period of the encountered waves. Mathematically, this phenomenon is described by the Doppler Shift which expresses that the *encountered wave frequency*, observed on a ship advancing relative to the inertial frame of reference, is different from the *absolute wave frequency*. Consequently, the Doppler Shift needs to be strictly taken into account when theoretical calculations of wave-induced responses are to be made since, otherwise, it is not possible to compare the result to any corresponding set of measurements. Note, in the remaining parts, deep-water conditions are assumed throughout. Moreover, all operational parameters, including vessel advance speed and encounter-wave angle, are constant.

It is a fact, see Section 2, that the Doppler Shift relates one - unique - absolute (wave) frequency to any one single encountered frequency when the velocity vectors of the ship and the waves have opposing projections on the ship's centreline, meaning that the angle between the pair of velocity vectors is larger than 90° and smaller than 270°; qualitatively expressed as the ship is in beam to head sea conditions. On the other hand, the Doppler Shift associates three absolute frequencies to any one single encountered frequency when two conditions are fulfilled, (1) the ship "follows" the progressive waves, (2) the encounter frequency takes a value less than a certain number, noting that the first condition implies that the angle between the pair of velocity vectors is either smaller than 90° or larger than 270° for the 1-to-3 relationship to be occurring.

The existing literature about the Doppler Shift, including its implications for ships sailing in waves, is fairly extensive when it comes to conceptual understanding and theoretical studies, see e.g. Lewandowski (2004); Price and Bishop (1974); Bhattacharyya (1978); Beck et al. (1989); Journée and Massie (2001). However, the situation is different when focus is on practical calculations related to real-case scenarios and applications. This is indeed so because of the 3-to-1 relationship, or mapping, between corresponding sets of frequencies in the absolute and the encounter domains when a ship advances in following waves[1]. Thus, any set of wave-induced measurements cannot be uniquely

* DTU Mechanical Engineering, Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark.
E-mail address: udn@mek.dtu.dk.

[1] While it may be a 3-to-1 mapping in the one "direction", it will be a 1-to-3 mapping in the other. Anyhow, the problem will generally from this point just be referred to as a *1-to-3* mapping problem.

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transferred, or transformed, into the absolute domain, if comparisons with theoretical calculations are to be made. Quite often this ”issue” is of no particular concern, since it is always possible, within theoretical calculations, to transform uniquely the other way, from absolute domain to encountered domain, when, or *if*, the wave conditions are known; although the encounter-wave spectrum may appear heavily distorted. In real-time, in-service scenarios, however, the wave conditions themselves need to be measured or estimated by *some* available means. This implies that the wave conditions need to be estimated from an *observation platform* that advances relative to the progressive waves, and, as a result, some practical approach to deal with the 1-to-3 relationship between encounter and absolute wave frequency must be introduced. As mentioned, there is a large literature related to the fundamental understanding and the conceptual, or theoretical, consequences of the Doppler Shift, whereas the practical complications are studied to a smaller degree. One interesting study, however, needs mentioning and, thus, it may be worthwhile to consult Lindgren et al. (1999) to obtain additional knowledge about the complications and consequences introduced because of the *stochastic* Doppler shift for ”a ship traveling with constant speed on a Gaussian random sea with a directionally distributed frequency spectrum”. This particular study (Lindgren et al., 1999) indicates, indeed, why it is by no means elementary to convert an encountered wave spectrum to a corresponding absolute one.

The present article deals specifically with a method to derive the absolute wave spectrum from an encounter-spectrum, representing the encountered distribution of wave energy spectral densities. As such, the work has its origin in studies related to the wave buoy analogy (Iseki and Ohtsu, 2000; Tannuri et al., 2003; Pascoal et al., 2007, 2017; Nielsen, 2006, 2008; Brodtkorb et al., 2018), where measurements of wave-induced vessel motions are processed to yield an estimate of the on-site (absolute) wave energy spectrum. Some of the studies consider vessels with zero-advance speed, while others, with smaller and larger degrees of success, introduce non-zero forward speed by establishing the mathematical equations directly in the absolute domain, and thus facilitating the wave energy spectrum as the very solution. However, one recent ship motion-based estimation algorithm (Nielsen et al., 2018; Nielsen and Brodtkorb, 2018) has proven computationally very efficient and capable in providing accurate estimates of the *encounter*-wave spectrum together with the relative wave heading angle for ships with advance speed. It is therefore highly relevant to address how to obtain the corresponding absolute wave energy spectrum.

In a previous work by Nielsen (2017), a practical approach was suggested for the transformation from encounter to absolute domain of wave spectral densities. The approach was tested comprehensively with numerical simulations, and the method has also been studied together with full-scale experimental data for its purpose in relation to ship motion-based wave spectrum estimation. Generally, the approach has been found to perform satisfactory but examples of the opposite also occur (Nielsen and Brodtkorb, 2018). The present work is introducing a second alternative transformation technique which relies on optimisation of a set of characteristic - absolute - wave parameters (e.g. significant wave height and zero-upcrossing period) based on an object function formulated through the wave spectral moments. This particular approach makes the transformed spectrum, in absolute domain, to have a shape of a parameterised wave spectrum such as a generalised JONSWAP spectrum (Hasselmann et al., 1973). It is the overall objective of this article to compare the new optimisation-based transformation technique with the ”old” transformation technique (Nielsen, 2017).

Besides the introduction, the article consists of five sections: Details and consequences of the Doppler Shift set the problem faced when spectrum transformation from encounter to absolute domain must be conducted, and this is explained in Section 2. The remedy in terms of a (new) specific transformation algorithm is given in Section 3, while application and testings on data are outlined in Sections 4 and 5 which contain performance evaluations, including comparative studies between the old and the new transformation techniques. Finally, Section 6 summarises and brings the conclusions and suggestions for further work.

2. Problem formulation

The distribution of wave energy density $S(\omega)$, or simply the wave spectrum, in a fixed point is considered, where ω is the absolute frequency. The wave energy in the infinitesimal range $d\omega$ is proportional to $S(\omega)d\omega$ and, as a result, the total energy of the wave system is proportional to $\int S(\omega)d\omega$. The distribution of wave energy density depends generally also on the physical location, say (x,y). However, in the present work, the variation with location is not considered, and throughout it is the distribution of wave energy density in a specific point which is addressed, noting that the point can be fixed or it can be advancing relative to the inertial frame of reference.

The Doppler Shift implies that the wave encounter-frequency, ω_e, is different from the absolute wave frequency ω_0, and it can be shown that the mathematical relationship (on deep water) is given by,

$$\omega_e = \omega_0 - \omega_0^2\psi, \;\; \psi = \frac{U}{g}\cos\chi \tag{1}$$

where U is the speed of the vessel, and χ is the angle between the pair of velocity vectors of the vessel and the waves, respectively. Note that indices $_0$ (absolute) and $_e$ (encounter) are used to emphasise the particular domains in question.

Although the vessel encounters waves at a frequency which is different from the absolute one, the total energy of the wave system is the same in the two domains, expressed as

$$E(\omega_e)d\omega_e = S(\omega_0)d\omega_0 \tag{2}$$

where $E(\omega_e)$ refers to the encountered distribution of wave energy density, while $S(\omega_0)$ is the corresponding one in absolute domain. Consequently, wave energy spectral density can, in theory, be easily transformed from the one domain to the other, simply by multiplication with the derivative, $\frac{d\omega_0}{d\omega_e}$ or $\frac{d\omega_e}{d\omega_0}$, depending on the ’transformation direction’;

$$E(\omega_e) = S(\omega_0)\frac{d\omega_0}{d\omega_e} \tag{3}$$

$$S(\omega_0) = E(\omega_e)\frac{d\omega_e}{d\omega_0} \tag{4}$$

As was indicated in the Introduction, wave spectrum transformation can be uniquely carried out when a ship sails ”against” the waves (beam to head sea). In certain conditions related to following seas[2], however, there exists no unique solution to the problem, since the Doppler Shift, Eq. (1), imposes a 1-to-3 relation between encounter frequency and absolute frequency, that is, one ’encountered wave component’ is the result of three (true) absolute wave components. Consequently, the energy density at the specific encounter frequency cannot be assigned uniquely to the corresponding set of absolute frequencies as illustrated in Fig. 1. This very problem has been comprehensively studied in Nielsen (2017), resulting in an elaborate, but at the same time practical, transformation algorithm. Briefly said, the algorithm is based on a scaling approach that assures preservation of energy (density) at corresponding sets of encounter and absolute frequencies by mapping the set of absolute frequencies into energy densities of a standard parameterised wave energy spectrum (e.g., Bretschneider or JONSWAP). Thus, a set of scaling ratios applies to specific absolute frequencies, obtained through the Doppler Shift of given encounter frequencies, and multiplication between, respectively, the scaling ratios, the encounter-

[2] Onwards, ”following waves” or ”following seas” are used to indicate a situation in which the 1-to-3 mapping problem occurs, without having a specific relative wave heading in study.