Ocean Engineering 165 (2018) 386–398

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Modeling of shallow water waves with variable bathymetry in an irregular domain by using hybrid finite element method

Prashant Kumar*, Rupali

Department of Applied Sciences, National Institute of Technology Delhi, Delhi, 110040, India

ARTICLE INFO

Keywords:
Mild slope equation (MSE)
Hybrid finite element method (HFEM)
Amplification factor
Numerical convergence
Pohang new harbor

ABSTRACT

An efficient numerical model is constructed with the combination of analytical and Hybrid Finite Element Method (HFEM) based on mild slope equation for shallow water waves to analyze the wave induced oscillation in an irregular geometrical domain. The domain of interest is divided in two regions as bounded and unbounded region. The solution of the mild slope equation is obtained in the bounded region with the consideration of variable bathymetry and partially reflected boundary using Hybrid element method. Further, an analytical approach is utilized in an unbounded region based on Fourier bessel series solution of scattered waves. Hybrid mesh elements is considered for bounded region to enhance the numerical accuracy. The numerical validation is conducted by the comparison of simulation results with analytical approximations and convergence analysis for the rectangular domain is also obtained. The amplification factor is obtained for T-shaped and TT-shaped domain to analyze the resonance modes. Finally, the current numerical model is applied on a realistic Pohang New Harbor (PNH) under the resonance conditions. Thus, the current numerical model can be utilized efficiently for redesigning and constructing artificial ports or harbors in the coastal regions with variable bathymetries.

1. Introduction

Harbor resonance is a major problem around the coastal zones caused by topographical variations, wave-wave interactions and interaction of waves with man made structures. Geometry of the harbor play significant role to generate local resonance. The excitation of harbor resonance due to wave propagation towards the coastal region may cause several difficulties such as loading and unloading of cargo process, damaging of coastal boundaries and breaking of mooring ropes etc (Kumar et al., 2016). It is difficult to restrict these incoming waves propagating towards the coastal regions, therefore, the mathematical model can be constructed based on diffraction, refraction and partial reflection of incident wave to analyze the resonance phenomenon in the coastal regions such as harbor, bays, canals etc. To analyze this problem, several linear and non linear numerical model have been employed based on Finite Difference Method(FDM) (Panchang et al., 1991; Madsen and Larsen, 1987; Nabavi et al., 2007), Boundary Element Method(BEM) (Kumar et al., 2015, 2013; Kumar and Gulshan, 2018; Cerrato et al., 2016, 2017), Finite Element Method(FEM) (Houston, 1981; Mei and Chen, 1976; Mercadé Ruiz et al., 2017) etc. Linear model are essentially based on Helmholtz equation (Kumar et al., 2014) and MSE whereas, non linear model are based on Boussinesq type equation (Gao et al., 2017a). Thus, estimation of wave impact around the coastal areas is significantly important in designing of harbors or modification in the existing ones.

In the past few decades, both experimental and theoretical studies on rectangular and circular shaped harbors were conducted on the problem of harbor resonance (Ippen and Goda, 1963; Hwang and Tuck, 1970; Lee, 1971). In addition, analytical studies has been performed on this problem by several investigators based on constant as well as varying water depth to analyze the wave oscillation in simple shaped harbors such as rectangular, circular (Ippen and Goda, 1963; Hwang and Tuck, 1970; Lee, 1971; Wang et al., 2015, 2014). The FDM is suitable for the problems with regular or simple geometries because it is quite easier to construct a mesh grid in the domain and code the problem (Panchang et al., 1991). For complex geometries, two dimensional (2D) BEM is utilized to solve Helmholtz equation for constant water depth problem (Kumar et al., 2015, 2013; Ippen and Goda, 1963; Hwang and Tuck, 1970; Lee, 1971; Kumar and Gulshan, 2017), whereas for varying bathymetries, the FEM can be employed to solve the shallow water wave problem (Tsay and Liu, 1989; Demirbilek and Panchang, 1998; Jung et al., 2016). Bellotti et al. (2012) introduced a technique which uses finite element method for shallow water waves for calculating the eigenvalues and eigenvectors of partially enclosed basins such as harbors and bays by applying a radiation condition at the interface boundary instead of zero surface elevation condition. A Galerkin

* Corresponding author.
E-mail address: prashantkumar@nitdelhi.ac.in (P. Kumar).

https://doi.org/10.1016/j.oceaneng.2018.07.024
Received 11 March 2018; Received in revised form 5 July 2018; Accepted 8 July 2018

FEM was introduced by Jung et al. (2016), which can be applied to regions where water depth approaches zero like coastal regions. Recently, several investigations have been done based on Boussinesq model to simulate the harbor resonance problem (Dong et al., 2013; Gao et al., 2018). Gao et al. (2016, 2017b) presented the non linear Boussinesq model for predicting the behavior of infragravity wave oscillations generated by group of incoming short wave in shallow water. The excitation of transverse oscillations within a harbor with small bottom slope was examined by Wang et al. (2013) and observed that these waves produces maximum amplitude at the backwall of the harbor and decreases in the offshore direction.

The propagation of water waves reproduced by MSE based on shallow water waves was first introduced by Eckart (1952) in early 1950s. After that, Berkhoff (1972, 1974) independently reformulated it for varying bathemetry and studied the combined effect of reflection, refraction and diffraction. Basically, MSE is developed for linear and non-linear waves, which are propagating over sea beds with mild slope. Further, many modification of the MSE have been introduced by various investigators for subsequent works (Kirby et al., 1992; Kirby, 1986; Massel, 1993; Beji and Nadaoka, 1957). Such as an extended version of the time dependent MSE was presented for both linear and non-linear waves which includes the effects of rapidly varying depth (Tong et al., 2010). Random modified MSE without using forcing terms was presented by Suh et al. (2001) and Zhang and Edge (1998). An improved or modified MSE using variational principle technique derived by Chamberlain and Porter (1995) describe the scattering properties of single and double ripple beds by including the higher order term that was not considered in Berkhoff MSE. It was reported that high order FEM for wave propagation problem shows accurate simulations for practical wave propagation analysis (Ham and Bathe, 2012; Schmicker et al., 2014). A model HARBD based on FEM has been used by the Army Corps of Engineers in various projects for harbor development to study infragravity waves in Barbers Point Harbor (Chen and Houston, 1987). Another approach HFEM was initially used by Berkhoff (1974) and hybrid formulation based on shallow water wave equation to analyze the harbor resonance problem was developed by Mei and Chen (1976). Hybrid functional proposed by Behrendt and Jonsson (1984) was also utilized. The hybrid element concept was followed to study wave scattering by offshore structures or islands (Houston, 1981; Tsay and Liu, 1989; Demirbilek and Panchang, 1998). Mixed and hybrid finite element method including convergence result and error estimates for second order elliptical problem was reported by Roberts and Thomas (1987).

In this paper, a novel hybrid numerical scheme is utilized based on the combination of analytical approximation(Fourier-Bessel expansion) and Finite element approximation with variable bathemetry to compute the wave amplification for the harbor with irregular geometries. In this formulation, the Fourier-Bessel expansion is used to solve the Helmholtz equation in unbounded region with constant depth and the Hybrid Finite element formulation is utilized in the bounded region with variable water depth. To enhance the numerical accuracy, hybrid mesh discretization is employed to provide the better convergence as compared to regular mess elements. The numerical scheme is also validated with previous well defined studies conducted by Ippen and Goda (1963), Lee (1971), Wang et al. (2015, 2011) for the rectangular shaped harbor. The present numerical scheme is implemented on T-shaped and TT-shaped harbor geometry with the consideration of partially reflecting boundaries to determine the wave characteristics in the domain of definition. The resonance modes of T-shaped and TT-shaped harbor are obtained and the wave field corresponding to these resonance modes is also analyzed. Additionally, present scheme is implemented on realistic Pohang New Harbor (PNH) (Kumar et al., 2014) and for this six locations based on the port stations are identified in PNH to analyze the wave amplification. The resonance mode at each recorder stations is also discussed. Further, the wave amplitude at different resonance mode inside PNH is also estimated to determine the safe location. Overall, the current numerical scheme provides an efficient tool to understand the resonance phenomenon in a highly irregular shaped harbor with variable bathymetry. Moreover, it can also be utilized for redesigning and constructing an artificial harbor.

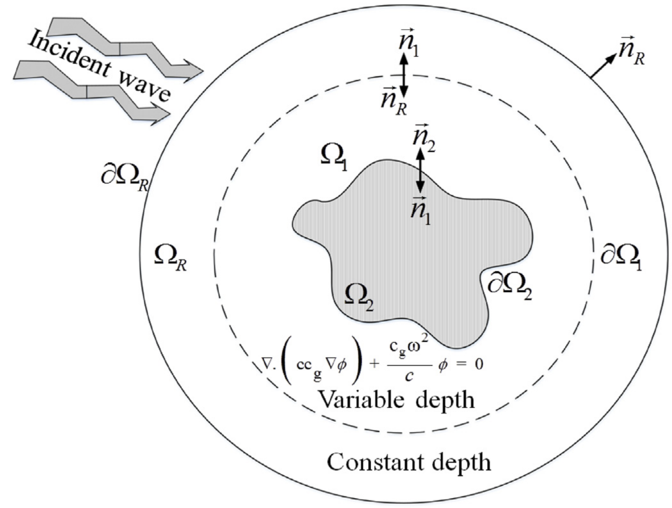

Fig. 1. Sketch of the model geometry including the bounded region Ω_1 with variable depth and the unbounded region Ω_R separated by a pseudo boundary $\partial\Omega_1$. The vectors $\vec{n}_1$, $\vec{n}_2$ and $\vec{n}_R$ represents the outward normal vector to region Ω_1, Ω_2 and Ω_R, respectively.

2. Mathematical formulation

2.1. Model geometry

The fluid domain is divided into bounded (Ω_1) and unbounded region (Ω_R). The boundary of Ω_R is lying at infinity and is denoted by $\partial\Omega_R$. It is assumed that wave scattering area Ω_2 is located inside the region Ω_1 with partially reflective boundary. An Pseudo boundary $\partial\Omega_1$ separates the region Ω_R and regions Ω_1 (see Fig. 1). Water depth is assumed constant in the region Ω_R and variable in region Ω_1. In Fig. 1 $\vec{n}_1$, $\vec{n}_2$ and $\vec{n}_R$ represents the outward normal vector to region Ω_1, Ω_2 and Ω_R, respectively.

In the region Ω_1, the MSE is given as follows

$$\nabla\cdot(cc_g\nabla\phi) + \frac{c_g\omega^2}{c}\phi = 0, \tag{2.1}$$

where $\nabla = (\partial/\partial x, \partial/\partial y)$ is the horizontal gradient operator, c_g and c are the group and phase velocity, k denotes wave number and $\phi(x, y)$ is the wave potential function. The higher order terms with bottom friction and entrance loss are neglected. The phase speed c is determined by the dispersion relation for small amplitude water waves, i.e, $\omega^2 = gk\tanh(kh)$, where g is the gravity acceleration, and $h = h(x, y)$ is the variable water depth. The group speed c_g is related to c by the given equation

$$c_g = \frac{c}{2}\left(1 + \frac{2kh}{\sinh(2kh)}\right). \tag{2.2}$$