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Cohesive crack propagation in a random elastic medium

M. Bruggi[a], S. Casciati[b], L. Faravelli[a,*]

[a] Department of Structural Mechanics, University of Pavia, via Ferrata 1, 27100 Pavia, Italy
[b] ASTRA Department, School of Architecture, University of Catania, via Maestranze 99, 96100 Siracusa, Italy

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Abstract

The issue of generating non-Gaussian, multivariate and correlated random fields, while preserving the internal auto-correlation structure of each single-parameter field, is discussed with reference to the problem of cohesive crack propagation. Three different fields are introduced to model the spatial variability of the Young modulus, the tensile strength of the material, and the fracture energy, respectively. Within a finite-element context, the crack-propagation phenomenon is analyzed by coupling a Monte Carlo simulation scheme with an iterative solution algorithm based on a truly-mixed variational formulation which is derived from the Hellinger–Reissner principle. The selected approach presents the advantage of exploiting the finite-element technology without the need to introduce additional modes to model the displacement discontinuity along the crack boundaries. Furthermore, the accuracy of the stress estimate pursued by the truly-mixed approach is highly desirable, the direction of crack propagation being determined on the basis of the principal-stress criterion. The numerical example of a plain concrete beam with initial crack under a three-point bending test is considered. The statistics of the response is analyzed in terms of peak load and load–mid-deflection curves, in order to investigate the effects of the uncertainties on both the carrying capacity and the post-peak behaviour. A sensitivity analysis is preliminarily performed and its results emphasize the negative effects of not accounting for the auto-correlation structure of each random field. A probabilistic method is then applied to enforce the auto-correlation without significantly altering the target marginal distributions. The novelty of the proposed approach with respect to other methods found in the literature consists of not requiring the a priori knowledge of the global correlation structure of the multivariate random field.

Keywords: Multivariate non-Gaussian random fields; Auto-correlation; Cohesive crack propagation; Truly-mixed finite-element method; Monte Carlo simulations

1. Introduction

The cohesive crack propagation problem is considered as a suitable example of having to generate non-Gaussian correlated random fields when considering the uncertainties of the physical parameters. The issue arises from observing that the simulation of non-Gaussian, multivariate random fields with a cross-correlation structure cannot be conceived but in an approximated manner [1]. Indeed, the task of matching the target marginal distributions conflicts with the one of preserving the spatial auto-correlation of each single-parameter random field. A probabilistic method for the generation of the non-Gaussian random fields is developed starting from the availability of traditional Gaussian random field realizations for each physical parameter, initially considered as independent of the others. In contrast to other methods found in the literature [2–4], the proposed procedure avoids the computational burden of directly considering the global correlation structure of the multivariate random field. In particular, the Gaussian realizations obtained by assigning each spectral density function are used to statistically estimate the covariance matrix of each corresponding random field. Thence, the auto-correlation structure of each random field is obtained in an already discretized manner, as an alternative to the common practice of first assigning an auto-correlation function of exponential type and then discretizing it [2]. The eigenvectors of each covariance matrix are then applied to the cross-correlated, non-Gaussian entries resulting from the inverse Nataf transform, in order to restore the auto-correlation structure of each field. Although the latter eigenvector mapping slightly alters the marginal distributions of the random

* Corresponding author.
E-mail address: lucia@dipmec.unipv.it (L. Faravelli).

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variables, the results of a sensitivity analysis show that accounting for the internal auto-correlation is fundamental in order to obtain reliable results in terms of statistics of the response.

The developed probabilistic method is first validated using a simple numerical example (whose results are reported in the Appendix), and is then applied to the problem of cohesive crack propagation. Three different fields are generated in order to model the spatial variability of the Young modulus, the tensile strength of the material, and the fracture energy, respectively, associated with the crack development. It is worth noting that a scalar representation of the Young modulus at any point of the body implicitly introduces an isotropy assumption, which is in conflict with the inherent anisotropic nature of a random medium. The finite-element discretization allows, however, to conceive an anisotropic medium as the assemblage of isotropic finite elements. Being a full simulation of all anisotropic elastic and failure parameters beyond the scope of this study, a scalar definition of the Young modulus is assigned at each point for the sake of convenience.

The numerical study of a crack-propagation phenomenon requires a mechanical model able to follow the crack-path evolution, which is a priori unknown and not aligned with the body discretization of the initial un-cracked domain. In [5], an adaptive remeshing strategy was proposed. Further studies aimed to limit and possibly avoid the computationally expensive remeshing phase. The procedures developed from these studies usually rely on either one of two alternative strategies: the XFEM (extended finite-element method), or the meshless approach. The XFEM method is based on a continuous displacement formulation which needs to be locally enriched with discontinuous modes in order to be able to cross the existing mesh by exploiting the "partition-of-unit property" of the shape functions [6]. The meshless strategy [7] seems to be ideally tailored to handle crack-propagation problems, but must overcome some numerical difficulties, such as the quadrature formulas and the boundary conditions assignment, which are easily solved by a finite-element approach.

The approach adopted in the present paper cannot be grouped in any of the two afore-mentioned categories, since it is based on an extension of the truly-mixed variational formulation developed in [8] from the Hellinger–Reissner principle. The associated solution algorithm, which is able to follow the a priori unknown crack path in both the pre- and post-peak regimes, was proposed and verified in [9]. This approach is chosen to be coupled with a Monte Carlo simulation scheme because it presents several advantages. By using equilibrated stress fields, with square-integrable divergence and inherently discontinuous displacements, the stress-flux continuity is imposed in an exact manner at each load step. Furthermore, the potentially active discontinuity of the displacements at each crack-interface element allows the direct inclusion of a cohesive law. Within this framework, the stress element of Johnson and Mercier [10] is selected, it being one of the very few elements able to pass the "inf–sup condition" required for the convergence of the method.

The problem of cohesive crack propagation in elastic media is investigated by coupling the above-mentioned mechanical model based on the truly-mixed formulation, and the newly proposed probabilistic method for the generation of 2D and multiparameter cross-correlated random fields of non-Gaussian nature. In the following, the theoretical backgrounds of the random field generation procedure and the truly-mixed finite-element formulation are discussed in separate sections, and are then jointly applied to a numerical example. Within this example, Monte Carlo simulations with finite elements are carried out to determine the statistics of the response of a plain concrete beam undergoing a three-point bending test. A sensitivity analysis is performed on a limited number of samples in order to preliminarily check the influence of different probabilistic assumptions on the results. Finally, the effects of the uncertainties on both the carrying capacity and the post-peak behaviour are quantified by considering a significant number of random field realizations.

The methodology developed in this work can be applied for further developments within fracture mechanics of quasi-brittle materials. Indeed, both the energetic size effect (of a deterministic nature) and the randomness in the material properties affect the maximum load-carrying capacity of a structural component. According to Ref. [11], for a certain class of structures, the first factor governs the deterministic mean of the nominal strength, while the second is responsible for the higher order moments. As such, a probabilistic approach is needed in order to evaluate the probability density function of the response, in view, for example, of an estimate of the reliability of the structure [12]. Furthermore, when considering the microscopic origin of the crack formation, homogenization techniques are usually applied to model a standard continuum which behaves like the originally micro-cracked body [13,14]. It is, therefore, of interest to check the hypothesis of a homogenized random field by evaluating the influence of the spatial variability of the material properties on the response.

2. The probabilistic approach

2.1. Framing the problem

In the literature, random fields were first introduced as a 2D natural extension of stochastic processes [15–17]. Moreover, the simulation of their realizations provided the support for the development of stochastic finite elements [18–21]. Advanced topics include the generation of spatial-temporal wind velocity fields [22,23] and the discretization issues in crack-propagation analysis [24]. An extended state-of-the-art report can be found in Ref. [25]. Refs. [1–4] are devoted to the development of simulation methods for non-Gaussian processes. Very few authors [2,4] discuss the simulation of multivariate and cross-correlated non-Gaussian random fields and the existing approaches are all, to some extent, approximate. For this reason, the issue of providing a non-Gaussian nature to a cross-correlated, multiparameter random field while preserving its internal auto-correlation structure, is still considered an open area of research.