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Semi-analytical solution for dynamic behavior of a fluid-conveying pipe with different boundary conditions

Xu Liang[a], Xing Zha[a], Xue Jiang[b,*], Lizhong Wang[a], Jianxing Leng[a], Zeng Cao[a]

[a] Ocean College, Zhejiang University, Hangzhou, Zhejiang, 310058, China

[b] Dept. of Naval Architecture, Ocean and Marine Engineering, University of Strathclyde, Henry Dyer Building, Glasgow, G1 5AE, UK

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ABSTRACT

This paper analyzes the dynamic behavior of a fluid-conveying pipe with different pipe end boundary conditions. The pipe is considered to be an Euler-Bernoulli beam, and a motion equation for the pipe is derived using Hamilton's principle. A semi-analytical method, which includes the differential quadrature method (DQM) and the Laplace transform and its inverse, is used to obtain a model for the dynamic behavior of the pipe. The use of DQM provides a solution in terms of pipe length whereas use of the Laplace transform and its inverse produce a solution in terms of time. An examination of the results of sampling pipe displacement at different numbers of sample points along the pipe length shows that the method we developed has a fast convergence rate. The frequency and critical velocity of the fluid-conveying pipe derived by DQM are exactly the same as the exact solution. The numerical results given by the model match well with the result obtained using the Galerkin method. The effect on pipe displacement of the pipe end boundary conditions is investigated, and it increases with an increase in the edge degrees of freedom. The results obtained in this paper can serve as benchmark data in further research.

1. Introduction

The vibration of a fluid-conveying pipe is a concern in many fields, such as marine engineering, aviation, construction machinery, chemical engineering, and oil exploration and refining. Thus improved knowledge of this phenomenon is widely applicable. A major risk for a free spanning pipeline is the failure caused by fatigue associated with various loads. The relatively large amplitudes of the oscillations of a free spanning pipe in vertical direction, caused by the external load, can lead to fatigue damage to the pipeline. Being able to accurately model the dynamic behavior of a fluid-conveying pipe is therefore important for theory development and has practical engineering significance.

The dynamic behavior of a fluid-conveying pipe became one of the hot topics since Brillouin (Paidoussis, 1998) first observed flow-induced vibration. Long (1955) first studied the vibration characteristics of a fluid-conveying pipe by experimental method, and observed that the natural frequency of the pipe decreases with the increase of the internal fluid velocity. Benjamin (1961a, 1961b) discussed the dynamic behaviors of a cantilevered fluid-conveying pipe, and found that flutter instability occurs in the pipe. Gregory and Paidoussis (1966a, 1966b) confirmed that the first unstable mode of the cantilevered fluid-conveying pipe is flutter instability instead of divergent instability through theoretical and experimental method, and investigated the effects of the mass ratio, damping and stiffness on critical velocity of instability. Chen (1970) investigated the forced vibration of a fluid-conveying pipe. Paidoussis and Issid (1974) gave the linear governing equation of the fluid-conveying pipe considering a variety of factors, and studied the vibration instability of the pipe under different boundary conditions. Paidoussis (1987) published a review article on the instability of cylindrical structures caused by fluid, which described the linear vibration of the pipe in detail, and pointed out two instability phenomena, namely flutter instability and buckling instability. More details on this subject can be referred in some monographs and reviews (Paidoussis, 1987; Paidoussis and Li, 1993). A variety of calculation methods have been proposed for the dynamic behaviors of the fluid-conveying pipe due to its wide application. Ibrahim (2010, 2011) and Li et al. (2015) have systematically summarized the progress of research into fluid-conveying pipe vibrations. Dai et al. (2014) studied a flexible pipe conveying fluctuating flows and analyzed the principal parametric resonances during lock-in for each of the first two modes using the direct perturbation method of multiple scales (MMS). He et al. (2017) enumerated the characteristics of vortex-induced vibrations in a pipe for the first two locked-in models under quasi-static displacement conditions using the Galerkin method. Li and Yang (2017) applied He's

* Corresponding author.
E-mail address: xue.jiang@strath.ac.uk (X. Jiang).

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variational iteration method (HVIM) to obtain the critical flow velocity and frequency of a fluid-conveying pipe under various boundary conditions. Zhai et al. (2011) analyzed the dynamic response of a Timoshenko pipeline (beam) under random excitation using the pseudo excitation method and the complex mode superposition method. Mohammadimehr and Mehrabi (2017) investigated the stability and free vibration of double-bonded micro-composite-sandwich cylindrical shells under magneto-thermo-mechanical loadings using the generalized differential quadrature method (GDQM). Kuiper and Metrikine (2005) used power series expansion and D-decomposition to study the stability of a vertically suspended free-hanging riser, and explained instability at small velocities of convection. Lin and Qiao (2008) used the differential quadrature method (DQM) to model the dynamical behavior of a fluid-conveying curved pipe subjected to motion-limiting constraints and harmonic excitation. Li and Yang (2014) obtained exact solutions for forced vibration of a fluid-conveying pipe using the Green function, and derived the natural frequencies of the fluid-conveying pipe. Chatzopoulou et al. (2016) investigated the vibration and instability of cyclically-loaded steel pipes during deep water reeling installation using the finite element method (FEM), and examined the effects of the modulus and the damping factor of the linear viscoelastic Winkler foundation and the fluid velocity on the resonance frequencies. Ni et al. (2011) used the differential transformation method (DTM) to analyze the vibration problem of a fluid-conveying pipe with several typical boundary conditions, and obtained natural frequencies and critical flow velocities for pipes. Hashemian and Mohareb (2016) suggested the finite difference model to analyze the sandwich pipes with thick cores subjected to internal and external hydrostatic pressure. Gu et al. (2016) modeled the dynamic response of a fluid-conveying pipe using a generalized integral transform technique (GITT), and analyzed the effect of aspect ratio on deflection and natural frequencies. Yazdi (2013) discussed the nonlinear vibration of doubly curved cross-ply shells using the homotopy perturbation method (HPM).

Comprehensive reviews of the calculation methods for vibration of fluid-conveying pipes are given in the above literature. However, a semi-analytical solution for the dynamic behavior of a fluid-conveying pipe has not yet been proposed. We propose a more accurate semi-analytical methodology that incorporates the differential quadrature method (DQM) and the inverse Laplace transform to investigate the dynamic behavior of a fluid-conveying pipe.

2. Method

2.1. Laplace transform and its numerical inversion

The Laplace transform is a linear transform which is widely used in structural dynamics. It can convert partial differential equations into ordinary differential equations or transform ordinary differential equations into algebraic equations (Liang et al., 2014). The Laplace transform and its inverse have been successfully applied to investigate the dynamic response of the structures subject to external load (Khalili et al., 2009; Lou and Klosner, 1973). Suppose f (t) is a real-valued function of time t, which defined in the real domain $[0, +\infty)$, the Laplace transform and its inversion are defined by:

$$\tilde{f}(s) = L[f(t)] = \int_0^{+\infty} f(t)e^{-st}dt \tag{1}$$

$$f(t) = L^{-1}[\tilde{f}(s)] = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} \tilde{f}(s)e^{st}ds \tag{2}$$

where L denotes the Laplace transform; L^{-1} denotes the inverse Laplace transform; s is a complex number in the Laplace domain; s_i (i = 1, 2, 3, …) are the singularities of $\tilde{f}(s)$; Re denotes the real part of the complex number. Let $s = \alpha + i\omega$, where α and ω are real numbers; the convergence condition of (2) can be stated as $\alpha > \alpha_0 = \max\,[Re(s_i)]$.

Durbin (1974) proposed that for the interval (0, T/2), the Laplace transform can be computed to any desired accuracy by the following formula (Liang et al., 2014):

$$f(t) \cong \frac{2e^{\beta t}}{T}\left\{\frac{\tilde{f}(\beta)}{2} + \sum_{n=1}^{N} \mathrm{Re}[\tilde{f}(\beta + n\pi i/T)]\cos(n\pi t/T)\right\} \tag{3}$$

where $\beta = 5/T$, $\mathrm{T} = 5 \times T_d$, T_d is the observing period in the time domain, and N is a large integer.

2.2. Differential quadrature method (DQM)

The differential quadrature method was first introduced into the field of structural dynamics by Bert and Malik (1996). In order to convert the derivative term with respect to the x coordinate to polynomials, the differential quadrature method (DQM) is employed to discretize the fundamental equations. Considering a continuous function $f(x)$, the nth order partial derivative with respect to x at a given point $x = x_i$ can be approximated by a linear weighted sum of function values at all the sample points in the domain of x, that is (Liang et al., 2015):

$$\frac{\partial^n f(x_i)}{\partial x^n} = \sum_{i=1}^{M} A_{ij}^{(n)} f(x_i)\ (i, j = 1,2, \ldots M) \tag{4}$$

where M is the number of sample points, and A(n) ij are the weighting coefficients of the nth order derivative defined by:

$$A_{ij}^{(1)} = \frac{\prod_{k=1,k\neq i}^{M}(x_i - x_k)}{(x_i - x_j)\prod_{k=1,k\neq j}^{M}(x_j - x_k)} \tag{5}$$

$$A_{ij}^{(k)} = k\left(A_{ij}^{(k-1)}A_{ij}^{1} - \frac{A_{ij}^{(k-1)}}{x_i - x_j}\right)(k = 1,2, \ldots M-1) \tag{6}$$

where $i, j = 1, 2, \ldots M$, but $i \neq j$, and the A(n) ii are defined as:

$$A_{ii}^{(k)} = -\sum_{j=1,j\neq i}^{M} A_{ij}^{(k)}\ (i = 1,2, \ldots M,\ k = 1,2, \ldots M-1) \tag{7}$$

3. Problem description

3.1. Governing equations

In this paper, we consider a linear elastic pipe of length L, which internally conveys an incompressible fluid with a velocity v and which is subjected to harmonic load q. The geometry and the Cartesian coordinate system are shown in Fig. 1, in which the origin of the (x, y, z) framework is considered to be located in space at the left end of the pipe (Chen, 1970; Dai et al., 2014).

The motion of the pipe in the vertical (y–axis) direction has been considered. In this paper, the following assumptions are made: (1) the fluid within the pipe (internal fluid) is incompressible and has a constant velocity; (2) the cross section of the elastic pipe is uniform, and the effects of shear, torsion, and rotational inertia can be neglected; and (3) the motion of the pipe is considered to be only in the vertical (y–axis) direction.

Adopting a small deformation assumption and treating the pipe as an Euler-Bernoulli beam, the equation for the motion of the pipe can be derived using Hamilton's principle. The kinetic energy of the system includes the kinetic energy of the pipe and the kinetic energy of the internal fluid, and is described by the equation:

$$T = \frac{1}{2}\int_0^l m_p\left(\frac{\partial w}{\partial t}\right)^2 dx + \frac{1}{2}\int_0^l m_f\left(\left(v\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right)^2 + v^2\right)dx \tag{8}$$

where w refers to the deflection in the vertical (y–axis) direction, m_p and m_f respectively denote the mass per unit length of the pipe and the internal fluid. The deformation energy of the system is