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Finite-order hydrodynamic model determination for wave energy applications using moment-matching

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ABSTRACT

The motion of a Wave Energy Converter (WEC) can be described in terms of an integro-differential equation, which involves a convolution product. The convolution term, which accounts for the radiation forces, represents a computational and representational drawback both for simulation, and analysis/design of control strategies. Several studies attempt to find a suitable finite parametric form that approximates the radiation impulse response, to express the equation of motion in the time-domain by a state-space representation. Ideally, this approximated parametric model should behave as closely as possible to the system under analysis, particularly at key frequencies, such as the resonant frequency of the device. This study presents a method to obtain a parametric model of both the force-to-motion dynamics and/or the radiation force convolution term, based on moment-matching. Recent advances in moment-matching, allow the computation of a model that exactly matches the frequency response of the original system at the chosen frequencies, while enforcing specific physical properties of the device, depicting a robust and efficient method to compute a state-space representation for the dynamics of a WEC. The potential of the algorithm is illustrated by numerical examples, and the approximation error is shown to be monotonically decreasing with increasing model order.

1. Introduction

Boundary Element Methods (BEM) are commonly used to calculate the hydrodynamic parameters of wave energy converters and, more generally, of various marine structures. While limited by the linear nature of potential flow theory, the speed with which numerical simulation may be performed when compared to other simulation methods, such as computational fluid dynamics or smoothed particle hydrodynamics, makes BEM a common choice to compute hydrodynamic parameters for a given WEC (Penalba et al., 2017a). Within the wave energy community, the most-widely used BEMs include the commercially available WAMIT (Newman and Lee, 2002) and the open-source NEMOH (Babarit and Delhommeau, 2015) numerical codes. However, one of the major drawbacks of BEMs is that the results are computed in frequency-domain and, hence, can only charaterise the steady-state motion of the WEC under analysis.

A more comprehensive dynamic modelling approach can be considered, using a time-domain representation of the motion of a WEC, in terms of the well-known Cummins' equation (Cummins, 1962). Moreover, a direct relationship between Cummins' equation and the hydrodynamic frequency-domain data (typically produced by WAMIT/NEMOH), is given in (Ogilvie, 1964) (see Section 3 for further details). The resulting time-domain dynamical model is an integro-differential equation, which contains a convolution term accounting for the fluid memory effects associated with radiation forces acting on a body.

Such a convolution operation usually represents a drawback, for two major reasons. Firstly, the direct computation of the convolution in a time-domain simulation scheme is computationally demanding. Secondly, such a term is inconvenient for the analysis and design of control systems, since modern (linear) control strategies are usually based on the availability of a state-space representation. Indeed, the vast majority of the optimal control techniques considered in the literature, which attempt to maximise the energy absorption of WECs, require a state-space approximation of the convolution term (Faedo et al.), with some notable exceptions, such as (Bacelli and Ringwood, 2015) and (Faedo et al.). This leads to the requirement for a suitable parametric approximation to the convolution term.

Several methods have been proposed in the literature to approximate the radiation convolution term, in terms of a linear time-invariant state-space representation. Noteworthy studies that provide a review on these multiple approximation methods, include (Taghipour et al., 2008), (Unneland, 2007) and (Roessling and Ringwood, 2015). These methodologies can be divided into two broad categories: time-domain and frequency-domain methods. A brief discussion on both approaches

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is given in the following.

Time-domain methods use impulse response data, which is usually generated (via the inverse Fourier transform) from the frequency-domain data computed by BEMs, mainly due to the computational effort required to compute the time-domain response directly. Studies that consider a time-domain formulation to obtain a state-space representation of the radiation convolution term include, for example, (Yu and Falnes, 1995), (Hatecke, 2015) and (Kristiansen et al., 2005). It is important to note that, in some studies (such as (Kristiansen et al., 2005)), an initial higher-order approximation is determined, followed by a model order reduction stage. In the particular case of (Kristiansen et al., 2005), model order reduction via balanced truncation (Antoulas, 2005) is considered for the second stage. An extensive discussion on this two-phase approximation procedure can be found in (Unneland, 2007).

Frequency-domain parameterisation methods attempt to compute a parametric model directly from the frequency-domain data calculated by BEMs. As discussed in (Taghipour et al., 2008), these methods can be divided into several categories. Some studies, such as (Sutulo and Soares, 2005) and (Xia et al., 1998), compute a parametric form for each hydrodynamic parameter (i.e. added mass and radiation damping) separately, and then reconstruct the corresponding radiation impulse response function. An alternative, and the most-widely used, formulation finds a state-space form for the radiation dynamics directly, based on its frequency response, which can be readily computed using the hydrodynamic characteristics of the device (see, for example, (Pérez and Fossen, 2008), (Jordán and Beltrán-Aguedo, 2004), (Holappa and Falzarano, 1998), (McCabe et al., 2005) and (Ø. et al., 2014)). A further alternative approach considered, for example, in (Perez and Lande, 2006), is to compute a state-space representation of the complete force-to-motion dynamics, instead of finding only a parameterisation of the radiation convolution term. In this case, the physical notion of each component of the state vector is somewhat lost, though the outputs still represent physical variables. Note that, with this overall formulation, the order of the state-space representation obtained is usually lower (for equal fidelity of the overall model) than first computing a parametric form for the convolution term separately, and then embedding it into Cummins' equation. In fact, this last approach always requires two additional elements in the state-space representation to describe the force-to-motion dynamics (i.e. position and velocity of the device). This difference between both methodologies can be of particular importance, for example, in model-based optimal control design for WECs, where an excessive number of model states can render an energy-maximising optimal controller unsuitable for real-time applications (the reader is referred to (Faedo et al.) for further details).

Regardless of the strategy chosen, a suitable parametric form, for wave energy applications, should represent either the force-to-motion dynamics or the radiation force convolution term (to incorporate into Cummins' equation), such that the behaviour of the approximated model is as close as possible to the target dynamics in a given (input) frequency range of interest. Furthermore, there are key frequencies, such as the resonant frequency of the device under analysis, that have a strong impact on the system dynamics. Ideally, the response of the approximated model should "match" the device dynamics at these specific key frequencies while, at the same time, approximating the behaviour of the target device over a frequency range of interest. Such a range is usually selected accordingly to the spectrum of the excitation force, as discussed in Section 4.1. Another important feature of a suitable identification technique is that the approximation error should decrease monotonically with increasing model order. This ensures that a higher number of elements to represent the state of the approximated model always decreases the approximation error. This is not always the case, as already reported in (Bertram et al., 2001), (Pérez and Fossen, 2008) and (Perez and Fossen, 2009) and can make the choice of approximating order somewhat haphazard. In particular (Pérez and Fossen, 2008; Perez and Fossen, 2009), report that the frequency-domain approximation algorithm studied suffers from stability issues when considering high-order approximations, although they declare that the approximation error will decrease significantly before expecting any increase in such an error value.

In light of the ideal characteristics described above, this paper proposes an approximation technique based on recent advances on model order reduction by moment-matching, developed over several studies, such as (Astolfi, 2010; Scarciotti and Astolfi, 2016a, 2017a, 2017b). As thoroughly discussed in Section 2, moment-matching methods are based on the idea of interpolating a certain number of points on the complex plane called *moments*. Moments have a direct relationship with the frequency-response of the dynamical system. In fact, a model reduced via moment-matching is such that its transfer function matches the behaviour of the transfer function of the target system at specific interpolation points (i.e. the moments). This is indeed one of the ideal features required in wave energy applications: a model, reduced by moment-matching, can be designed to match *exactly* the frequency response of the device under analysis, at specific key frequencies. Such an approach has several advantages compared to an identification plus reduction technique (as considered, for example, in (Kristiansen et al., 2005)): there is no need to perform a higher order identification of the system, since the reduced order model matches the moments of the unknown system, it is not just the result of a low-order identification but it actually retain some key properties of the system under analysis (Scarciotti and Astolfi, 2017b). Furthermore, given this intuitive property of the moment-matching approach, essential physical properties of the device can be enforced on the reduced order model, such as input-output stability. Note that stability is not usually guaranteed by current radiation force impulse response identification algorithms, so that several "fixes" have been proposed (further discussed in Section 5).

We note that the process of determining a finite-order dynamical model from 'frequency response' data points can alternatively be termed system identification (determining a model from frequency response data), or model-order reduction, where the starting model order is effectively the number of frequency points available. In the paper, we use the term 'model order reduction', in order to be more consistent with previous literature on moment-matching.

The remainder of the paper is organised as follows. In Section 2, the definition of *moment*, and the theoretical framework behind model order reduction by moment-matching, is introduced. Section 3 recalls the equation of motion of a floating body, in both frequency and time domain formulations. In Section 4, moment-based model order reduction is applied to the WEC case, to obtain a suitable parametric form for both the complete force-to-motion dynamics, and just the convolution term of Cummins' equation. Section 5 presents numerical examples of the proposed technique, using frequency-domain data for particular WEC devices. Finally, a discussion and concluding remarks are presented in Section 6.

1.1. Notation and preliminaries

Standard notation is considered through this study, with any exceptions detailed in this section. $\mathbb{R}^+$ ($\mathbb{R}^-$) denotes the set of non-negative (non-positive) real numbers. $\mathbb{C}^0$ denotes the set of pure-imaginary complex numbers and $\mathbb{C}^-$ denotes the set of complex numbers with a negative real part. The symbol 0 stands for any zero element, dimensioned according to the context. The symbol $\mathbb{I}_n$ denotes an order n identity matrix. The spectrum of a matrix $A \in \mathbb{R}^{n\times n}$, i.e. the set of its eigenvalues, is denoted as $\sigma(A)$. The symbol $\oplus$ denotes the direct sum of n matrices, i.e. $\oplus_{i=1}^{n} A_i = \text{diag}(A_1, A_2, ..., A_n)$. The notation $\Re\{z\}$ and $\Im\{z\}$, with $z \in \mathbb{C}$, stands for the *real-part* and the *imaginary-part* operators, respectively. The expression $||x||_2$, with $x \in \mathbb{C}^{n\times 1}$, denotes the ℓ^2-norm of the complex-valued vector x. The *Kronecker product* between two matrices $M_1 \in \mathbb{R}^{n\times m}$ and $M_2 \in \mathbb{R}^{p\times q}$ is denoted as $M_1 \otimes M_2 \in \mathbb{R}^{np\times mq}$, while the convolution between two functions $f(t)$ and $g(t)$