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Empirical deep-water heave added-mass and radiation-damping expressions for 2-D rectangular-sectioned floating bodies

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ABSTRACT

Two-dimensional added-mass and radiation damping data of Vugts (1968) related to heaving rectangular sections in deep water are represented by empirical formulas. Although other data exist, the Vugts data are considered to be the standard in experimental planar-motion hydrodynamics research of basic geometries of many floating bodies, including those of several wave energy conversion systems. The present empirical effort was undertaken to make the Vugts results useful in strip-theory analyses of the heaving and pitching motions of slender vessels having rectangular sections. For approximate predictions of the hydrodynamic coefficients (added-mass and radiation damping) of these systems, the empirical expressions provide manageable analytical expressions. In the derivations, the behaviors of the parametric constants for each coefficient are found to be approximate rectilinear functions of the beam-to-draft ratio. The applicability of the empirical expressions is enhanced by this finding.

1. Introduction

The prediction of wave-induced motions of slender barge-shaped bodies can be performed with good accuracy by using the *strip-theory* - a two-dimensional theory. Slender bodies are those for which the beam and draft are much smaller than the vessel length. The strip-theory has its foundation in fluid mechanics, and its application to floating bodies was first done by Krylov (1896), according to Pedersen (2000). The strip-theory was revised over a half-century later by Korvin-Kroukovsky (1955). The method, as presented by that author, was later corrected and refined by Korvin-Kroukovsky and Jacobs (1957). The work of these authors was further modified by Motora (1964). Those referenced papers describe the application of the strip-theory to the planar motions of ships in regular seas. The theory was expanded by Salvesen et al. (1970). That work is considered to be the cornerstone of contemporary ship-motion analysis using the strip theory. An excellent review of the linear strip-theory is presented by Bishop and Price (1979). There have been several formulations of *nonlinear strip theories*. One of the earliest of these is the “quadratic theory” of Jensen and Pedersen (1979). More recently, Fan and Wilson (2004) present a time-domain non-linear strip theory. The basics of strip theory are described by McCormick (2010, 2014). Other advances in the strip theory include those of Journée (1992) and Bandyk and Beck (2009).

Of interest herein are the deep-water two-dimensional hydrodynamic inertial and damping force terms in the equations of motion. The former force term is due to the inertial reaction of the ambient water to body motion; while, the latter is due to the body-motion induced surface waves that carry energy from the body. The most accurate method available to determination of these two-dimensional hydrodynamic forces is that incorporating boundary elements and numerical solution techniques. See, for example, the paper of Koo and Kim (2015). The applications of numerical techniques are somewhat expensive, depending on the software package used. In this paper, empirical expressions for rectangular sections are presented to represent the frequency-dependent added-mass and radiation damping coefficients of the equations of motions. Furthermore, the expressions include the beam-to-draft ratio effects on the hydrodynamic coefficients. This aspect greatly expands the applicability of the equations.

2. Empirical and experimental added-mass and radiation damping coefficients

The prediction of the performance of a rectangular barge system depends, to a great extent, on the expressions for the added-mass and radiation damping coefficients in the equations of motion. Consider the rectangular barge section sketched in Fig. 1. Although not shown, the

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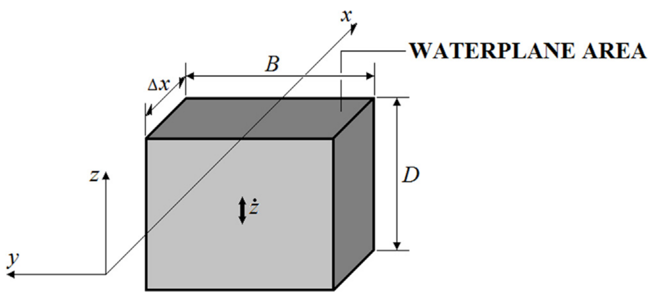

Fig. 1. Notation and sketch of a rectangular strip undergoing heaving oscillations.

barge has a length (L) that is much greater than both the beam (B) and draft (D). The barge, then, can be considered to be a hydrodynamically *slender body*. By doing so, the strip-theory can be used with rather good accuracy. The error associated with the use of (two-dimensional) strip theory is not significant if the slender-body conditions are satisfied, as discussed by Newman (1977). The experimentally determined two-dimensional hydrodynamic coefficients of Vugts (1968) are directed at the use of the strip-theory.

2.1. Added-mass coefficient

Consider, first, the dimensionless added-mass coefficient (μ), defined as the ratio of the added-mass and the displaced mass - both, per unit body length. Vugts (1968) presents results of this coefficient as a function of the deep-water wave number (k_0) parameter, defined as

$$\zeta \equiv \omega\sqrt{\frac{B}{2g}} = \sqrt{k_o\frac{B}{2}} \tag{1}$$

The deep-water wave number is $k_o = 2\pi/\lambda_o$, where λ_o is the deep-water wavelength. Also, in eq. (1) are the gravitational acceleration (g), the uniform beam (B) of the section and the circular frequency of the heaving motion (ω). The experimental data of Vugts (1968) for this coefficient over the dimensionless wave number range of approximately 0.5 #ζ = $(k_0B/2)^{1/2}$ # 1.75 are presented in Fig. 2. For $\zeta > 0.35$, it appears that the data can be represented by the following expression:

$$\mu \equiv \frac{m_w{}'}{\rho BD} = \mathrm{A+B}J_0(\mathrm{C}\zeta^n) \tag{2}$$

Here, m'_w is the added-mass per unit length of the barge section, A, B, C and n are to-be-determined parametric constants, D is the barge draft and ρ is the mass-density of the displaced water. The prime (') is used to indicate "per unit length". The function J_0 ( ) in eq. (2) is the Bessel function of the first kind, zero-order. The Vugts data in Fig. 2 are found to have one minimum point and one inflection point. It is these points upon which, in part, this empirical analysis is based. Let the value μ_1 correspond to the minimum point in a data set at ζ_1. At that point, the slope of the μ-ζ curve, then, equals zero ($d{:}/d\zeta|_1 = 0$). The inflection point is that at which the second-derivative (curve) is zero ($d^2{:}/d\zeta^2|_2 = 0$). In addition to these points, we choose the next point where $d\mu/d\zeta|4 = 0$, and specify that the infinite-frequency value, μ4, is at that point. For a given B/D value, μ4 is approximately obtained from the formula derived by McCormick and Hudson (2008), which is the following:

$$\mu_\infty = \frac{1}{2\pi}\frac{B}{D}\sqrt{\left[\left(4\frac{D^2}{B^2}\right)\ln\left(4\frac{D^2}{B^2}\right) + \left(1 - 4\frac{D^2}{B^2}\right)\ln\left(1 + 4\frac{D^2}{B^2}\right) + 8\frac{D}{B}\tan^{-1}\left(\frac{B}{2D}\right)\right]^2 + \pi^2} \tag{3}$$

Values obtained from this equation for the Vugts (1968) experiment are presented in Fig. 2.

As stated, four conditions are required to determine the four parametric constants in eq. (2). Referring to Vugts (1968) data in Fig. 1, the conditions used here are as follows:

a. The slope of the μ-ζ curve is zero at ζ_1, where $\zeta_1 > 0$. Mathematically, the condition is $d\mu/d\zeta|_1 = 0$. In order to satisfy this condition, $J_1(\mathrm{C}\zeta_1{}^n) = J_1(\mathrm{Q}_1) = 0$, where the argument of the Bessel function is

$$\mathrm{C}\zeta_1^n \equiv \mathrm{Q}_1 \simeq 3.8317 \tag{4}$$

Note: If $n - 1 < 0$, the ζ_1 can equal 0; however, that condition does not result in the behavior of Vugts' data. The numerical value is approximately that given by Abramowitz and Stegun (1965) and others.

b. The approximate value of μ_1 at ζ_1 is obtained from the data in Fig. 2a, as illustrated in Fig. 2b.
c. The second derivative of the μ-ζ curve is zero at ζ_2, where $\zeta_2 > \zeta_1$, illustrated in Fig. 2b. Mathematically, we write $d^2\mu/d\zeta^2|_\zeta 2 = 0$. Although the ζ_2-value is a little difficult to identify for the data in Fig. 2, it can be identified with an error of about ± 0.05 for the $B/D = 2$ and 4 data. Hence, these data are used to determine the argument of the Bessel function in eq. (2). Using the notation introduced in eq. (4), the Bessel function argument is

$$\mathrm{Q}_2 \equiv \mathrm{C}\zeta_2^n \tag{5}$$

which is unknown. Combine equations (4) and (5) by eliminating C and, then, solve for n. The result can be combined with the condition needed to satisfy the condition $d^2\mu/d\zeta^2|_\zeta 2 = 0$ to obtain the following equation:

$$n = \frac{1}{\mathrm{Q}_2}\frac{J_1(\mathrm{Q}_2)}{J_0(\mathrm{Q}_2)} = \frac{\ln(\mathrm{Q}_1/\mathrm{Q}_2)}{\ln(\zeta_1/\zeta_2)} \tag{6}$$

The values of point coordinates (ζ_1, μ_1, ζ_2 and μ_2) are found to be approximate rectilinear functions of B/D. These functions are the following:

$$\zeta_1 \simeq 0.0817\left(\frac{B}{D}\right) + 0.4768 \tag{7}$$

$$\mu_1 \simeq 0.3167\left(\frac{B}{D}\right) + 0.1667 \tag{8}$$

$$\zeta_2 \simeq 0.1067\left(\frac{B}{D}\right) + 1.1467 \tag{9}$$

and

$$\mu_2 \simeq 0.4000\left(\frac{B}{D}\right) + 0.3000 \tag{10}$$

Curves resulting from equation (7) through (10) are presented in Fig. 3.

The coefficients A and B in eq. (2) are evaluated by simultaneously solving the following equations resulting from the respective applications of eq. (2) to points 1 and 2:

$$\mu_1 \simeq 0.3167\left(\frac{B}{D}\right) + 0.1667 = \mathrm{A+B}J_0(\mathrm{Q}_1) \tag{11}$$

Here, the approximation of μ_2 is from eq. (10). The value of Q_1 is in eq. (4); while, the Q_2 values are in Table 1. The results obtained from solving equations (10) and (11) for A and B are in Table 2 with the calculated values of C resulting from either equation (3) or (4). Again, the results of the empirical dimensionless added-mass coefficient are presented in Fig. 2 with the measured values of Vugts (1968).