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A Bayesian Monte Carlo-based algorithm for the estimation of small failure probabilities of systems affected by uncertainties

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ABSTRACT

The estimation of system failure probabilities in presence of uncertainties may be a difficult task when the values involved are very small, so that sampling-based Monte Carlo methods may become computationally impractical, especially if the computer codes used to model the system response require large computational efforts, both in terms of time and memory. In this work, we propose to exploit the Bayesian Monte Carlo (BMC) approach to the estimation of definite integrals for developing a new, efficient algorithm for estimating small failure probabilities. The Bayesian framework allows an effective use of all the information available, i.e. the computer code evaluations and the input uncertainty distributions, and, at the same time, the analytical formulation of the Bayesian estimator guarantees the construction of a computationally lean algorithm. The proposed method is first satisfactorily tested with reference to an analytic, two-dimensional case study of literature, offering satisfactory results; then, it is applied to a realistic case study of a natural convection-based cooling system of a gas-cooled fast reactor, operating under a post-loss-of-coolant accident (LOCA), showing performances comparable to those of other efficient alternative methods of literature.

1. Introduction

Reliability and risk analyses are fundamental tasks for the economical sustainability and the safety of many complex engineering applications [1]. In the last decades, they have developed into scientific disciplines, as an answer to the increasing needs of mass production and potentially highly hazardous systems, e.g. complex structures, nuclear power plants, chemical plants, radioactive waste repositories, etc.

In all generality, the problems of risk and reliability arise whenever it is possible to identify a potential source of damage or loss, i.e. hazardous events threatening some targets of interest. In this context, a central role is played by uncertainty. In general, uncertainties in the operation of an engineering system are due to some lack of knowledge, e.g. not fully understood physical phenomena underlying its functioning, measurement errors or scarce operating experience to characterize it, errors and approximation in the mathematical models and the computer codes used to represent its behaviors, etc. Unfortunately, these uncertainties may give rise to functional failures, i.e. deviations from their modeled, expected behavior which lead the "loads" imposed to the systems to overcome their "capacities". The estimation of the probabilities of these failures is a fundamental task, which, in general, requires the propagation of the uncertainties, described by properly identified probability density functions (pdfs), to the model outputs with respect to which some system performance indicator is defined [2–5]. The propagation is typically done by classical, crude Monte Carlo (MC) schemes, which are based on the repeated runs (or simulations) of the computer codes representing the system behavior in correspondence of different sets of the uncertain input values sampled from their joint pdf [1]. In details, given a probabilistic model of the input/output mapping of the system under analysis, described by a n-dimensional random vector $\boldsymbol{x} = (x_1, ..., x_n)$ with probability density function $f(\boldsymbol{x})$, and a performance function $H(\boldsymbol{x})$ representing some system response of interest, the failure event and its indicator function are defined as

$$F = \{\boldsymbol{x} : H(\boldsymbol{x}) \leq 0\} \tag{1}$$

$$1_F(\boldsymbol{x}) = 1_{\{H \leq 0\}}(\boldsymbol{x}) \tag{2}$$

where the set F is called failure region. The failure probability can, then, be written as

$$p_f = P(\{\boldsymbol{x} : H(\boldsymbol{x}) \leq 0\}) = \mathrm{E}_f[1_F(\boldsymbol{x})] = \int_{\mathrm{R}^n} 1_F(\boldsymbol{x}) f(\boldsymbol{x}) d\boldsymbol{x} \tag{3}$$

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where the set of points $\boldsymbol{x} \in \mathrm{R}^n$ such that $H(\boldsymbol{x}) = 0$ is the limit state function [6].

The classical MC scheme [1] to the estimation of the failure probability then amounts to i) sampling N_{calls} values of the input parameters $\boldsymbol{x}$ from $f(\boldsymbol{x})$ and ii) running the model in correspondence of each of the N_{calls} realizations of $\boldsymbol{x}$ in order to compute the performance function $H(\boldsymbol{x})$; the failure probability can, then, be estimated by dividing the number of realizations for which $H(\boldsymbol{x}) \leq 0$ by N_{calls}.

However, the engineering systems involving potentially large economical and/or safety issues are designed so that their failures are rare events. Thus, the estimation of the failure probabilities can require a large number of runs of the computer codes encoding the systems behaviors. This problem is worsened by the fact that the complexity of the systems involved and the levels of detail and accuracy typically required by the analyses are such that the computer codes become computationally very intensive, as, for example, the finite elements codes required in structural reliability analyses [11,7].

Various efficient methods have been proposed in literature to address this problem: the interested reader may refer to [9,10] for thorough reviews and comparisons of many existing method. However, efforts are still made by the scientific community aimed at improving existing methods or developing new ones, mainly because their performances depends in general on the models and/or computer codes adopted to carry on the reliability analysis, and there not exists an approach with outstanding performances in any application. Here, we will briefly recall the general ideas behind the most recent methods. The first family of methods commonly used in structural reliability analysis and known as FORM or SORM (first or second order reliability methods), stems from an approximation of the limit state function around the so called "most probable failure point (MPFP)" or "design point", based on a Taylor series expansion [6]. The fast estimates of the failure probabilities, requiring a very limited number of performance function evaluations by the original model comes at the expense of a few important limitations: i) the methods do not allow any quantification of the approximation errors; ii) in case of complicated, highly non-linear limit state functions, the linear approximation provided by the FORM introduces large estimation errors, only partially reduced if the SORM is used; iii) in presence of multiple, non-connected failure domains the methods may lead to biased estimates of the failure probability; iv) when dealing with high dimensional input spaces, the finite difference scheme may severely affect the efficiency of these methods.

The second family of methods, also known as simulation methods, comprises those based on MC schemes. In this context, many so called variance reduction techniques have been proposed in literature, which aim at developing more efficient MC estimators achieving the same levels of accuracy at largely reduced numbers of model simulations, i.e. evaluations of the performance function. Perhaps, the most popular variance reduction technique is Importance Sampling (IS), which has been successfully applied in many fields of research. However, it is not easy in general to choose a suitable importance density from which the samples should be drawn; a common approach in structural reliability is that of choosing the importance density as a joint Gaussian distribution centered around the MPFP identified by a FORM (or SORM) in the isoprobabilistically transformed standard input space [11,12]: by doing so, it is possible to refine the result of the FORM (SORM) by an IS procedure which picks the samples in the vicinity of the failure region. Another important variance reduction technique is subset sampling, which estimates the failure probability as the product of conditional probabilities, each of them being estimated by Markov Chain Monte Carlo simulation [13]. The benchmark study provided in [10] showed that the subset sampling (or modifications thereof) method is very effective in higher dimensions. In general, sampling-based methods stemming from a variance reduction technique allows significant improvements with respect to a crude MC simulation; however, they suffer from the fact that the number of time-demanding evaluations of the original performance function required for estimating small probabilities remains too large [7].

The third family of methods for efficiently addressing this problem relies on the substitution of the original performance function by a surrogate model (or metamodel) within a sampling-based scheme; a metamodel is, in general, orders of magnitude faster to be evaluated, thus allowing significant computational savings. Several metamodels have been proposed in literature, such as quadratic response surfaces, polynomial chaos expansions, support vector machines, neural networks and kriging. The major drawback of the direct substitution of the original performance function with a surrogate model is that it is often impossible to keep the approximation error under control [7].

Recently, adaptive strategies for coupling sampling-based method and metamodeling have been proposed, which allow refining the metamodel construction until a predefined level of accuracy is achieved. For example, [7] proposed to resort to a kriging-based surrogate model to approximate the optimal importance density; following a different philosophy, [14,11] proposed to use kriging to reconstruct the limit state function, whereby the metamodel training set was, then, iteratively enriched on the basis of a learning function accounting for the probability of the metamodel correct classification (AK-MCS and its improved version, AK-IS). The meta-IS [7] turns out to be more suitable than the AK-IS algorithm [11] when the failure region is made up of disconnected sub-regions, each one providing non negligible contributions to the total failure probability: in this case, in fact the AK-IS provides biased estimates, since it relies on a FORM estimate of the MPFP. In a work by some of the same authors [15], the AK-IS has been successfully extended to be able to deal with multiple disconnected failure regions. A drawback of both IS-based and metamodel-based approaches (as the AK-IS) is that they suffer when applied to highly multidimensional input spaces, as acknowledged in [16,17].

In this work, we propose to exploit the Bayesian Monte Carlo (BMC) approach to the estimation of definite integrals [18] for developing a new, efficient algorithm for estimating small failure probabilities. The motivation of the proposed strategy lies in the fact that the BMC method has been shown in [18] to be capable of offering better performances than crude MC and even of importance sampling (IS)-based variance reduction techniques, both in terms of accuracy and required number of model evaluations, by a more efficient use of the available information, i.e. the input pdfs and the computer code runs. Moreover, the BMC method features (i) a closed-form analytic expression, which may become a key advantage when the computational times required by the estimation algorithms (not the model evaluations) start affecting the overall failure probability estimation, and (ii) the possibility of a priori selecting the evaluation input points, which is useful, for example, when exploiting an existing set of model evaluations, obtained in the past for different purposes [18]. These considerations suggest the possibility of exploiting the BMC approach for estimating the special definite integrals which define the failure probabilities (see Eq. (3)).