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Dynamic response of free span pipelines via linear and nonlinear stability analyses

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ABSTRACT

A thorough stability analysis of submerged pipeline dynamics has been performed. The problem of current-induced vibrations in a free spanning pipeline was analyzed using both linear and non-linear methodologies. While most investigations dedicated to the analysis of this type of problem use either approximate analytical methods or traditional numerical methods, two alternative approaches are presented: Linear Stability Analysis (LSA) and the Generalized Integral Transform Technique (GITT). Although strictly valid when the pipeline is subject to small amplitude disturbances, LSA is simple to apply and yields accurate either explicit or transcendental analytical relations that predict dominant pipeline vibration frequencies and wavelengths as well as the onset of buckling. On the other hand, the GITT can predict pipeline behavior for arbitrary disturbance amplitudes more efficiently than traditional numerical methods. These features are illustrated by considering the problem of a submerged pipeline subjected to soil conditions and ocean currents. This problem was solved using LSA, the GITT and the Finite Element Method (FEM) for comparison purposes. Moreover, the DNV-RP-F105 standard was also used for verifying the proposed methods. The alternative methods offered in this paper can overcome limitations associated with traditional analytical and numerical approaches, and are employed to produce previously unreported results.

1. Introduction

Free spans are commonly encountered in underwater pipelines that lay on the ocean bottom due to a frequently uneven seabed. This particular configuration in association with ocean currents and soil stiffness can greatly accentuate fatigue failure. As a result, frequency analyses become a crucial part of pipeline design for ensuring pipeline integrity and preventing expensive maintenance. Nowadays, the DNV-RP-F105 standard (DNV-RP-F105, 2006) has been used as major industry guideline in configurations involving free spanning pipelines. In spite of the relevance of this international standard, it is limited to a number of free span scenarios, and in many cases alternative methods are required.

One common approach to derive the dynamic response of free spanning offshore pipelines is to use approximate analytical methods. These are often based on a simplified beam theory and consequently of easy application (Fyrileiv and Mork, 2002). Approximate methods are still in use, as presented in (Sollund et al., 2015a), where closed-form analytical expressions for the fundamental frequency of short free spans were obtained using dimensional analysis. The inherent simplicity of these type of methods has drawbacks, since their approximate character can lead to inaccurate solutions, limiting their application to simpler problems.

On the other end of the spectrum, traditional numerical schemes such as Finite Element Methods (FEM) (Huges, 1987; Zienkiewicz et al., 2013), are capable of solving extremely complex problems while still providing good accuracy. Consequently, these methods have been widely used for analyzing subsea pipeline dynamics (Forbes and Reda, 2013; Santos et al., 2014). Naturally, these methods often require a significant amount of CPU time to meet strict accuracy requirements.

Combining the advantages of numerical and analytical approaches, semi-analytical methods arise as a powerful alternative to the former ones. They can provide improved accuracy for complex nonlinear problems, as compared to analytical methods, while still consuming a reasonably small amount of CPU time when compared to traditional numerical schemes. When looking into free span problems, there are some literature studies that employ methods of this kind. (Vedeld et al., 2013) presented a semi-analytical solution for the harmonic response of pipelines, based on the Rayleigh-Ritz approach in combination with a displacement field taken as a Fourier series. In subsequent studies, the dynamic response of both free spanning pipelines (Sollund et al., 2015b), as well as multi-span pipelines (Sollund et al., 2014), was

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analyzed using the same methodology.

Two well-established semi-analytical methods that are particularly interesting for the analysis of underwater pipeline dynamics are the Linear Stability Analysis (LSA) and the Generalized Integral Transform Technique (GITT). The LSA methodology provides the asymptotic response of dynamical systems when their steady-states are subjected to small disturbances. This is achieved by transforming the original nonlinear governing equations into a simpler linear and modal form that is then numerically solved. In spite of this simplification the method can still provide highly accurate solutions and be applied to complex problems. Most LSA applications involve fluids mechanics problems, such as convective and absolute asymptotic instabilities (Huerre and Monkewitz, 1990), transient disturbance growth due to non-monotonic dynamic behavior (Schmid, 2007) and the evaluation of the stability of dynamical systems with two and three-dimensional steady-states (Theofilis, 2011). There are, however, studies related to fluid-structure interaction problems as well, as seen in (Dowell and Hall, 2001).

While LSA involves the solution of a simpler model derived from the original problem formulation, the GITT (Cotta, 1993) addresses the original model in its complete form, in which the sough solutions are expressed as series of orthogonal eigenfunctions. This hybrid analytical-numerical technique is akin to the FEM, as the solutions are written in terms of basis functions. However, the usage of orthogonal bases leads to solutions with higher accuracy. Furthermore, it requires no domain discretization, many times leading to smaller CPU times. The GITT has found numerous applications in heat and fluid flow over the past decades. More recently, it has also been applied to a variety of dynamical systems, such as wind-induced vibrations in overhead conductors (Matt, 2009), dynamics of axially moving beams (An and Su, 2011, 2014a), dynamics of damaged structures (Matt, 2013), vibration in orthotropic plates (An and Su, 2014b), pipe dynamics in the presence of two-phase flow (Gu et al., 2013; An and Su, 2015) and general one-dimensional distributed systems (Matt, 2015).

It is important to highlight that the combined application of Integral Transforms with Linear Stability Analysis has been previously employed. This was first presented in studies focused on the onset of natural convection in porous cavities (Alves et al., 2002) and in superposed fluid and porous layers (Hirata et al., 2006). The former employed a linearization of the integral-transformed system, whereas the latter linearized the original model prior to the integral transformation processes. This second alternative, which involves solving the differential eigenvalue problem resulting from the application of LSA using the GITT, has also become common in recent years (Sphaier and Barletta, 2014; Sphaier et al., 2015; Alves and Barletta, 2015).

Although there are a few literature studies that employ generalized integral transforms to problems involving mechanical systems, there are no studies related to the application of the GITT to free spans in submerged pipelines, let alone combined GITT-LSA applications to these types of problems. In this context, the main purpose of this work can be summarized as two-fold: *i*) propose LSA as an alternative to approximate analytical methods capable of providing accurate solutions to complex problems while maintaining simplicity of use and small CPU time requirements, and *ii*) propose GITT as an alternative to traditional discretization methods that can simulate nonlinear dynamical systems with lower CPU time requirements while still providing highly accurate solutions to complex problems. These are demonstrated as feasible alternatives through the analysis of the dynamic response of a free span pipeline on an uneven seabed that is subjected to axial force, soil stiffness and seawater movement. Numerical results are then compared to the DNV-RP-F105 standard (DNV-RP-F105, 2006), and new data for configurations not covered by this standard are provided.

2. Mathematical model

Consider a pipeline laying on a seabed subjected to soil conditions and weak ocean currents, as sketched in Fig. 1 (a), and the corresponding physical model illustrated in Fig. 1(b), where T is the axial force, L is the pipeline length, k is the soil stiffness and (x, y) are the spatial coordinates.

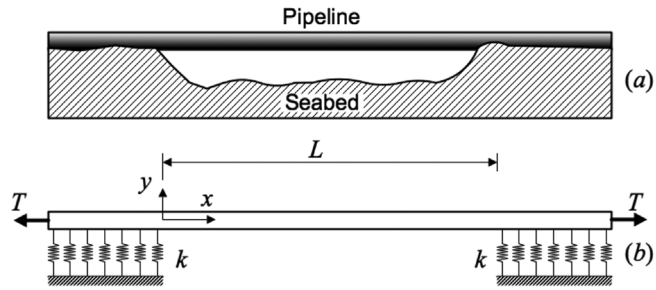

Fig. 1. Seabed (a) pipeline and (b) physical model.

The governing equation for the considered model is given by

$$-EI\frac{\partial^4 y}{\partial x^4} + T\frac{\partial^2 y}{\partial x^2} + \frac{\rho_w C_d D}{2}\left|u_c - \frac{\partial y}{\partial t}\right|\left(u_c - \frac{\partial y}{\partial t}\right) + \left(\frac{\pi}{4}\rho_m D^2 C_m a_c + f'_w\right) = \left(\frac{\pi}{4}\rho_w D^2 C_a + m'\right)\frac{\partial^2 y}{\partial t^2}, \tag{1}$$

where E is Young's modulus, I is the moment of inertia, ρ_w is the seawater density, ρ_m is the pipeline density, C_d is the drag coefficient, and the outer and inner diameters of the pipeline are D and D_i, respectively. Moreover, u_c and a_c are the ocean water current velocity and acceleration, whereas $m' = \pi(D^2 - D_i^2)\rho_m/4$ is and $f'_w = m'g$ are the mass and weight per unit length. Finally, C_m is an inertia coefficient, C_a is an added mass coefficient, g is the gravity acceleration and $c = \sqrt{EI/(L^2(\pi\rho_w D^2 C_a/4 + m'))}$ is the phase speed of a disturbance with wave length L in a pipeline with zero axial force. These parameters and variables are then rewritten in dimensionless form using

$$\tau = \frac{tc}{D},\quad \xi = \frac{x}{L},\quad \eta = \frac{y}{D},\quad \beta = \frac{L^2}{D^2},\quad \alpha^2 = \frac{TL^2}{EI},\quad \kappa = \frac{kL^3}{EI},$$
$$\gamma = \frac{L^4}{DEI}\left(\frac{\pi}{4}\rho_m D^2 C_m a_c + f_w'\right),\quad U = \frac{u_c}{c}\quad \text{and}\quad \delta = \frac{\rho_w c^2 C_d L^4}{2EI}, \tag{2a-j}$$

such that Eq. (1) can be re-written in dimensionless form as

$$-\frac{\partial^4 \eta}{\partial \xi^4} + \alpha^2\frac{\partial^2 \eta}{\partial \xi^2} + \delta\left|U - \frac{\partial \eta}{\partial \tau}\right|\left(U - \frac{\partial \eta}{\partial \tau}\right) + \gamma = \beta\frac{\partial^2 \eta}{\partial \tau^2}. \tag{3}$$

One important remark about this equation must be pointed out: only cases with $U \ll 1$ can be considered because the pipeline-current interaction model used in Eq. (3) assumes that the ocean current exerts a force on the pipeline that is proportional to the pipeline transversal velocity. Hence, no vortex induced vibrations (VIV) are herein considered. This imposes $\mathrm{Re} = \rho_w\, u_c\, D/\mu_w < 48$ as an approximate upper limit for the maximum value of the Reynolds number of the ocean flow around the pipeline, where μ_w is the dynamic viscosity of ocean water. Nevertheless, one should point out that this constraint is only approximate, because it is strictly valid for a two-dimensional straight pipeline.

Different pipeline-soil interactions are analyzed using distinct combinations of boundary conditions. A first scenario considers a generalized pinned condition where the soil stiffness allows vertical motion proportional to the shear force, but there is still no bending moment at the boundaries:

$$\frac{\partial^3 \eta}{\partial \xi^3} = -\kappa\eta \quad \text{and} \quad \frac{\partial^2 \eta}{\partial \xi^2} = 0 \quad \text{at} \quad \xi = 0, \tag{4a}$$

$$\frac{\partial^3 \eta}{\partial \xi^3} = \kappa\eta \quad \text{and} \quad \frac{\partial^2 \eta}{\partial \xi^2} = 0 \quad \text{at} \quad \xi = 1. \tag{4b}$$

A second configuration considers a generalized clamped condition where the soil stiffness also allows vertical motion proportional to shear force, but there is still no slope at the boundaries: