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Analytical representation of nonlinear Froude-Krylov forces for 3-DoF point absorbing wave energy devices

Giuseppe Giorgi*, John V. Ringwood

Centre for Ocean Energy Research, Maynooth University, Maynooth, Co. Kildare, Ireland

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ABSTRACT

Accurate and computationally efficient mathematical models are fundamental for designing, optimizing, and controlling wave energy converters. Many wave energy devices exhibit significant nonlinear behaviour over their full operational envelope, so nonlinear models may become indispensable.

Froude-Krylov nonlinearities are of great importance in point absorbers but, in general, their calculation requires an often unacceptable increase in model complexity/computational time. However, for axisymmetric bodies, it is possible to describe the whole geometry analytically, thereby allowing faster calculation of nonlinear Froude-Krylov forces.

In this paper, a convenient parametrization of axisymmetric body geometries is proposed, applicable to devices moving in surge, heave, and pitch. While, in general, Froude-Krylov integrals must be solved numerically, by assuming small pitch angles, it is possible to simplify the problem, and achieve a considerably faster algebraic solution. However, both nonlinear models compute in real-time.

The framework presented in the paper offers flexibility in terms of computational and fidelity levels, while still representing important nonlinear phenomena such as parametric pitch instability. Models with lower computational requirements may be more suitable for repetitive calculations, such as real-time control, or long-term power production assessment, while higher fidelity models may be more appropriate for maximum load estimation, or short-term power production capability assessment.

1. Introduction

Mathematical models are indispensable for designing, optimizing, and controlling wave energy converters (WECs). Ideally, such models are required to be both accurate and computationally efficient. The most popular models are linear, which are convenient for their short computation time, but accurate only for small relative fluid/body motions. Conversely, wave energy converters are likely to experience large movements, especially under controlled conditions, in order to maximize the power absorption. Consequently, significant nonlinear effects may arise, so that linear models become less reliable (Giorgi and Ringwood, 2017c).

The inclusion of nonlinear terms in the equation of motion generally improves the accuracy of the model, but with additional complexity and computational burden. In particular, it has been shown, in the literature, that nonlinear Froude-Krylov (FK) forces, which represent the integral of the static and dynamic pressure over the wetted surface of the device, are especially important for point absorbers (Giorgi and Ringwood, 2017a). Furthermore, nonlinear FK forces are responsible for purely-nonlinear phenomena, such as pitching instability or parametric roll (Tarrant, 2015).

For geometries of arbitrary complexity, the computation of nonlinear FK forces first requires the discretization of the surface with a mesh, and then the employment of a time-consuming remeshing routine, at each time step, in order to calculate the instantaneous wetted surface of the device (Matusiak, 2011; Bandyk, 2009). However, if the body is assumed to be axisymmetric, it is possible to describe the complete geometry analytically, thereby avoiding the use of a mesh (Giorgi and Ringwood, 2017b). Note that such a hypothesis is not particularly restrictive, since the vast majority, if not all, point absorbers are designed to be non-directional, and are therefore axisymmetric. Due to the analytical description of the geometry, the computation of nonlinear FK forces is considerably faster than the meshing approach.

In this paper, a convenient parametrization of axisymmetric surfaces is proposed, applicable to devices moving in three degrees of freedom (DoFs), allowing an analytical description of nonlinear FK forces in surge, heave, and pitch. In general, the FK integrals must be solved numerically using, for example, a trapezoidal rule. Assuming

* Corresponding author.
E-mail address: giuseppe.giorgi.2015@mumail.ie (G. Giorgi).

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small pitch angles, however, it is possible to further simplify the problem, and achieve an algebraic solution, which is considerably faster than numerical integration. Hereafter, LFK is used as the acronym for *linear* FK forces, VFK for the algebraic-nonlinear model (since it assumes a *vertical* geometry), and RFK for the numerical-nonlinear model (since it considers a *rotating* geometry).

The application of these three different approaches to the FK force calculation (linear, algebraic-nonlinear, and numerical-nonlinear) may depend on the particular purpose the mathematical model is intended for. For preliminary studies, shape optimization, or WEC farm configuration analysis, many iterations are required; therefore, the requirement for fast calculation prevails over the accuracy requirement. In case of control optimization routines, a higher level of accuracy is of great importance, at a significantly low computational time; therefore, the algebraic-nonlinear approach may be the most appropriate. Finally, higher degree of accuracy is needed for power production assessment, or to compute maximum loads (for the design of the mechanical properties of the structure and mooring lines), or for verifying the likelihood of events such as instability or parametric roll. In such cases, the numerical-nonlinear method may be preferred.

However, the choice between the LFK, VFK, and RFK, strongly depends on the operational space spanned by the device in its operating conditions, in particular the heave displacement and the pitch angle. In fact, for small motions, linear assumptions are reasonably valid, and all models effectively overlap. Conversely, when the device experiences large motions, typically induced by the control strategy, important differences between the models may arise.

The purpose of this paper is to provide a simple and computationally convenient formulation for nonlinear FK forces for axisymmetric wave energy converters, moving in surge, heave, and pitch. A case study is then considered, inspired by the CorPower device (CorPower, 2017), in order to quantify differences in accuracy and computation time for linear, algebraic-nonlinear and numerical-nonlinear models. Previous studies, in the literature, concerning nonlinear FK forces for multi-DoF wave energy devices, use a computationally expensive mesh-based approach (Penalba et al., 2017), while this paper introduces a more efficient methodology, applicable to point absorbing WECs.

Although such a modelling approach for nonlinear FK forces calculations was already proposed in (Giorgi and Ringwood, 2017b), only very academic case studies were considered, such as spheres, while this paper demonstrates the applicability of the method to a *real device*; this allows a realistic quantification of the computational efforts related to a geometry composed of different elementary geometries, as well as the discussion of more realistic nonlinear effects. A further novelty of this paper is the expansion of the method to *multiple DoFs*, as opposed to 1-DoF as in (Giorgi and Ringwood, 2017b). In fact, two solutions are proposed and discussed (VFK or RFK), with significantly different computational burdens (about two orders of magnitude difference). The choice between algebraic (VFK) or numerical (RFK) integrations is guided by the accuracy/computational compromise, specific to the particular application the model is intended to serve. Furthermore, the expansion to 3-DoFs is not trivial, especially for the numerical integration solution (RFK): some practical issues are here addressed, leading to two different approaches, one of which is almost twice as fast as the other.

The reminder of the paper is organized as follows: Sect. 2 presents the different methods to compute FK forces, which are validated in Sect. 3. A parametric study is proposed in Sect. 4, while the dynamical response to incoming waves is discussed in Sect. 5. Some final remarks and conclusions are given in 6. An appendix is included as well, in order to explicitly provide all the algebraic results, obtained with the VFK model.

2. Froude-Krylov forces

In the framework of linear potential theory, FK forces correspond to the integral of the pressure of the undisturbed wave field over the wetted surface of the device. Such a pressure is defined, according to linear Airy's theory, as:

$$p(x, z, t) = p_{st} + p_{dy} = -\gamma z + \gamma a \frac{\cosh(\chi(z+h))}{\cosh(\chi h)}\cos(\omega t - \chi x + \varphi) \tag{1}$$

where $p_{st} = -\gamma z$ is the static pressure, p_{dy} the dynamic pressure, γ the specific weight of the sea water, a the wave amplitude, χ the wave number, ω the wave frequency, φ an arbitrary phase (usually set to zero), h the water depth (defined according to a right-handed inertial frame of reference (x, y, z), with the origin at the still water level (SWL)), x pointing in the direction of propagation of the wave, and z pointing upwards. The free surface elevation η is defined as

$$\eta(x, t) = a\cos(\omega t - \chi x + \varphi) \tag{2}$$

Froude-Krylov forces are computed by integrating the pressure, shown in equation (1), over the instantaneous wetted surface $S(t)$. In particular, static and dynamic FK force components can be defined, respectively, as follows:

$$\mathbf{F}_{FK_{st}} = \mathbf{F}_g + \iint_{S(t)} -\gamma z \mathbf{n} dS \tag{3a}$$

$$\mathbf{F}_{FK_{dy}} = \iint_{S(t)} p_{dy} \mathbf{n} dS \tag{3b}$$

where $\mathbf{n} = (n_x, n_y, n_z)$ is the unit vector normal to the surface, pointing outwards, and $\mathbf{F}_g$ is the gravity force. Likewise, FK torques are defined as follows:

$$\mathbf{T}_{FK_{st}} = \mathbf{r} \times \mathbf{F}_g + \iint_{S(t)} -\gamma z \mathbf{r} \times \mathbf{n} dS \tag{4a}$$

$$\mathbf{T}_{FK_{dy}} = \iint_{S(t)} p_{dy} \mathbf{r} \times \mathbf{n} dS \tag{4b}$$

where $\mathbf{r}$ is the position vector, and × is the cross product.

For a geometry of arbitrary complexity, it is not possible to solve the FK integrals of (3a) to (4b). Linear boundary-element solvers linearize the problem around the still water level (SWL: $z = \eta = 0$), therefore considering a constant wetted surface, unlikely to be valid for a WEC under energy-maximizing control (Giorgi and Ringwood, 2017c).

Alternatively, the geometry can be discretized through a mesh, computing the contribution to the force over each mesh panel (Gilloteaux, 2007). Such an approach, though feasible, is computationally expensive, due to the recalculation, at each time step, of the instantaneous wetted surface, and consequent remeshing of the geometry. For *axisymmetric* buoys, a convenient parametrization of the wetted surface can ease the calculation of the FK integrals. In particular, computationally efficient algebraic solutions of the FK integrals exist for vertical axisymmetric buoys (Giorgi and Ringwood, 2017b). Such a method is further described in Sect. 2.1.

If the body is also pitching, numerical integration is required. Such a method is further described in Sect. 2.2. Table 1 summarises the main different characteristics of LFK, VFK, and RFK, highlighting different assumptions and, qualitatively, different computational time requirements. Quantitative accuracy and computational time comparisons are presented in Sects. from 3 to 5.

2.1. Nonlinear Froude-Krylov force: algebraic integration

Both the algebraic (VFK) and the numerical (RFK) integration approaches rely on the assumption of axisymmetric geometry, which allows the analytical description of the whole wetted surface. The geometry of a generic buoy which is symmetric around a vertical axis can be described in cylindrical coordinates, as follows:

$$\begin{cases} x(\rho, \theta) = x_G + f(\rho)\cos\theta \\ y(\rho, \theta) = f(\rho)\sin\theta \qquad , \quad \theta \in [-\pi, \pi) \wedge \rho \in [\rho_1, \ \rho_2] \\ z(\rho, \theta) = z_G + \rho \end{cases} \tag{5}$$