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# Estimation of significant wave period from wave spectrum

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ABSTRACT

Although the significant wave period is one of the key parameters in the design of coastal structures, it is not calculated by the spectral wave model because it is obtained by the zero-crossing analysis. The present paper examines several formulas that relate the significant wave period to wave spectral parameters such as peak period, mean period, and peakedness parameter. The formulas are derived based on the wave measurements in two locations in the Japan/East Sea (JES). The derived formulas are then compared with the measured significant wave periods in other locations in JES. It is shown that the formula using the mean wave period, $T_{m-1,0}$, is the most accurate. This formula is further used in a spectral wave model, the significant wave periods from which are compared with the measurement. Additionally, the relationship between the significant wave height and period is obtained based on the wave hindcasting, from which the most probable significant wave period can be determined for a specific significant wave height. A comparison with the measurement shows that the relationship is relatively inaccurate in the locations where southerly swell waves are significant, which are not accurately taken into account in the numerical model.

## 1. Introduction

The zero-crossing and spectrum analyses give the wave parameters but wave heights and periods from the two methods are different to each other. However, assuming the narrow-banded process, the significant wave height from the spectrum analysis, $H_{m0}$, is equivalent to that of zero-crossing analysis, $H_{1/3}$. Additionally, the average zero-crossing period, , can be expressed in terms of the spectral moments, which is $T_{m02} = (m_0/m_2)^{1/2}$ (Bitner-Gregerson and Magnusson, 2004; Cartwright and Longuet-Higgins, 1956; Longuet-Higgins, 1952; Thornton and Guza, 1983). Here the $n$-th order moment of wave spectrum is $m_n = \int_0^\infty f^n S(f)df$. Due to the equivalence in the mean wave periods, Cherneva et al. (2008) have compared the mean wave period from the wave hindcasting to the measurement.

On the other hand, even though the significant wave period from the zero-crossing analysis, $T_{1/3}$, is used in the design of coastal structures (Goda, 2010), the spectral wave model cannot predict the significant wave period. For this reason, several researchers tried to estimate the significant wave period with the wave spectral parameters. Among them, Takahashi et al. (1979) proposed an empirical formula, $T_{1/3}/T_{m02} = 1.1$. Similarly, Suh et al. (2010) predicted the significant wave period with the formula, $T_{1/3}/T_{m02} = 1.14$. Meanwhile, Goda (2010) stated that $T_{1/3}/T_p$ was similar to $T_{m-1,0}/T_p$. Here $T_p$ denotes the peak wave period, and $T_{m-1,0}$ is defined as $m_{-1}/m_0$, which is provided as the mean wave period in the WAM model (Günther et al., 1992). In addition to this, Goda (2010) related the ratio $T_{1/3}/T_p$ to the peak enhancement factor of JONSWAP (Joint North Sea Wave Observation Project) wave spectrum, $\gamma_J$, and the power of frequency in Huang et al.'s (1981) Wallops wave spectrum, $m_w$. However, since not all wave spectra fit to JONSWAP or Wallops wave spectrum, Goda's (2010) formula cannot be applied to all wave spectra. Besides the accuracies of the previously stated formulas have not been compared to each other, despite of its importance.

For this reason, several formulas on the significant wave period from the wave spectral parameters, $T_s$, are proposed in this study, which were derived from two wave measurements in the Japan/East Sea (JES). The formulas are compared with other wave measurements in JES. After that, the most accurate one is then applied to Chun and Ahn's (2017) wave hindcasting to improve the accuracy in the significant wave periods. Furthermore, the relationship between the significant wave height and period in the JES is established based on the wave hindcasting.

## 2. Formulas for significant wave period

### 2.1. Wave measurements in the Japan/East Sea (JES)

In the present study, the formulas on the significant wave period are

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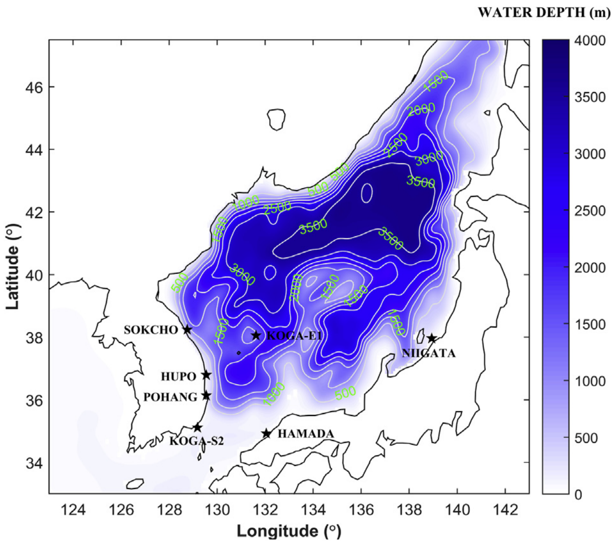


**Fig. 1.** Location of wave measurements.

examined using the wave measurements in the JES (Fig. 1). Due to this, the wave spectral parameters should be given but they are not available in all the measurement stations. In particular, the wave measurements at Pohang, Hamada, and Niigata do not provide the wave spectral parameters. For this reason, the wave data at each station were used for different purposes. Of the wave measurements, the three-dimensional wave displacements are given at KOGA-E1 and KOGA-S2, which were measured with the wave buoys having the sampling frequency of 2 Hz. By applying the zero-crossing and FFT (Fast Fourier Transform) analyses to the wave displacements, the significant wave period and wave spectral parameters were obtained. Then three formulas for the significant wave period were derived in terms of the wave spectral parameters. On the other hand, the wave measurements at Hupo and Sokcho also provide the wave spectral parameters. However, the waves at these stations have been measured by the bottom-mounted wave equipment, which has problems such as signal attenuation. The accuracy of the bottom-mounted wave equipment is not so good as that of buoy equipment, even though the equipment has been improved in these days. Accordingly, the wave data at these stations were used to validate the formulas without being included in the derivation of the formulas. Meanwhile, the significant wave periods measured at Pohang, Hamada, and Niigata, where the wave spectral parameters are not given, are compared with the computed values from Chun and Ahn's (2017) wave hindcasting model.

### 2.2. Estimation of significant wave period based on wave spectral parameters

In the present study, the ratio, $T_{1/3}/T_p$, is expressed as a function of $Q_p$, as follows:

$$\frac{T_{1/3}}{T_p} = a(Q_p - 1)^b \tag{1}$$

where $Q_p$ denotes Goda's (1970) peakedness parameter. Its form was given as $Q_p = (2/m_0^2)\int_0^\infty fS^2(f)df$. Since $Q_p$ can be evaluated for any wave spectrum and it is linearly related to $\gamma_J$ (see Appendix), it is employed in Eq. (1). The coefficients in Eq. (1) were estimated as $a = 0.8$ and $b = 0.08$, respectively, by the least square method. Fig. 2(a) shows that the significant wave period calculated by Eq. (1) well agrees with the measurement. The accuracy of Eq. (1) is presented in Table 1 in terms of several statistical parameters such as bias, root mean square error (RMSE), scatter index (SI), Pearson's correlation coefficient, $r$, and Willmott's (1981) index of agreement. Of the statistical quantities, the descriptions on bias, RMSE, SI, and $r$ are given in Pilar et al. (2008). Here the bias is calculated by subtracting the predicted value from the measurement. Therefore, the minus sign in the bias indicates overestimation of the prediction formula. The Pearson's correlation coefficient measures the linear relationship between two data (Hirsh et al., 1993), and it is more sensitive to outliers than to observations near the means (Legates and McCabe, 1999). On the other hand, the Willmott's index of agreement measures the degree to which the formula's predictions are error-free but not the correlation between prediction and measurement. It varies between 0 and 1.0, where 1.0 indicates perfect agreement and 0 connotes complete disagreement.

Meanwhile, as shown by Suh et al. (2010) and Takahashi et al. (1979), the ratio, $T_{1/3}/T_{m02}$ is almost constant. From the wave measurements, it is written as

$$\frac{T_{1/3}}{T_{m02}} = 1.12 \tag{2}$$

Fig. 2(b) shows that the significant wave period calculated by Eq. (2) is somewhat scattered and underestimated. As Eq. (2) is equal to the average of the formulas of Suh et al. (2010) and Takahashi et al. (1979), it was compared with those formulas (Fig. 2(b)). While the RMSEs of Takahashi et al. (1979) and Suh et al. (2010) are 0.65 s and 0.63 s, respectively, the RMSE of Eq. (2) is 0.62 s. On the other hand, the statistical parameters in Table 1 indicate that the prediction by Eq. (2) is worse than that by Eq. (1).

The significant wave period, $T_{1/3}$, is also related to the mean wave period, $T_{m-1,0}$. According to Fig. 3(a), $T_{m-1,0}$ is highly correlated with $T_{1/3}$, but it is 0.47 s larger than $T_{1/3}$ on average. Accordingly, the relationship between $T_{1/3}$ and $T_{m-1,0}$ was assumed as

$$T_{1/3} = cT_{m-1,0}^d \tag{3}$$

where the coefficients $c$ and $d$ were estimated as 0.76 and 1.11, respectively, again by the least-square method. Fig. 3(b) compares the significant wave period calculated by Eq. (3) with the measurement, showing that the predicted values well agree with the measurement and the scattering is much smaller than other formulas. Table 1 also shows that Eq. (3) is more accurate than other formulas even though it slightly more overestimates than Eq. (1).

### 2.3. Validation of the formulas using other wave measurement data

The formulas derived in the previous section are validated by comparing the calculated significant wave periods with the measurements at Hupo and Sokcho, which were not used in the derivation of the formulas. Fig. 4 shows a comparison between the predicted and measured significant wave periods, and the accuracy of the prediction formulas are given in Table 2. All the values of bias in Table 2 are positive, indicating that all the formulas underestimate the significant wave period, as can be seen in Fig. 4 as well. However, Eq. (3) gives the smallest bias. Other statistical parameters also indicate that Eq. (3) is more accurate than other formulas.

Based on the results given in Tables 1 and 2, it is concluded that Eq. (3) is the most accurate formula to predict the significant wave period from wave spectral parameters. This formula is, therefore, used to calculate the significant wave period in the wave hindcasting model described in the next section. On the other hand, the RMSE of the significant wave period by Eq. (3) for the wave data in Tables 1 and 2 is 0.52 s, and the average significant wave period is 5.79 s. From this, the uncertainty of Eq. (3) is estimated as 8.9%. In addition to this, since $T_p$ is also used as the reference wave period, the significant wave period by Eq. (3) is compared with $T_p$ in this study. Fig. 5 shows that $T_s$ is 1.38 s smaller than $T_p$ on average. $T_{1/3}$ is also smaller than $T_p$ but the average value of $T_{1/3}/T_p$, 0.87, is larger than that of $T_s/T_p$, 0.80, due to the underestimation in the significant wave period by Eq. (3).