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A probabilistic approach for the quantification of prediction error in deterministic phase-resolved wave forecasting

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ABSTRACT

This paper presents a semi-analytical methodology for the determination of prediction error statistics in deterministic sea wave predictions (DSWP), based on linear wave models. The underlying wave elevation is modelled as a Gaussian stochastic process and the coefficients of the wave propagation model are assumed to be determined by linear fitting on available measurements in time and/or space. The possible data contamination due to measurement error is also explicitly considered. The resulting approach eventually provides a Linear Estimator of Prediction Error (LEPrE) in time and space, in terms of prediction error standard deviation, given the fitting procedure and the sea spectrum. The presented approach allows supplementing deterministic predictions based on phase-resolved linear wave models with a sound prediction error measure, and allows defining the concept of "Predictability Region" in a consistent probabilistic framework. Example applications are reported, both for long-crested and short-crested waves, with verification through Monte Carlo simulations. Single point wave gauge/wave buoy measurements as well as wave radar measurements have been considered as simulated examples. The developed methodology is also compared with existing approaches highlighting and discussing both the differences and the interesting qualitative commonalities.

1. Introduction

The nowadays interest about deterministic wave propagation models based on the marine wave radar technology is encouraged by the outlook of possible applications to real-time waves and ship motion forecasting. The development of early-warning, guidance and decision-support systems based on deterministic prediction procedures could possibly have a positive impact for the safety and operability at sea. A main asset in this kind of emerging short-term forecasting technology (with temporal horizon of the order of minutes, and spatial horizon of the order of hundreds meters) is the marine wave radar. In fact, the marine wave radar has been shown to be potentially capable of scanning the sea surface and retrieving the instantaneous images of the nearby wave field in a wide spatial range (Dankert and Rosenthal, 2004; Nieto Borge et al., 2004; Serafino et al., 2011; Naaijen and Wijaya, 2014). It is however to be noted that challenges in modelling of the associated basic electromagnetic backscattering mechanism still require evolutions of this technology to obtain very accurate measurements, at least when used to feed deterministic sea wave prediction (DSWP) models. The LIDAR technology has also been explored for the measurement of wave elevation (Belmont et al., 2007; Grilli et al., 2011; Nouguier et al., 2014). In principle LIDAR could be considered as an alternative to wave radar. However, presently, available research on corresponding local wave elevation measurements (Belmont et al., 2007; Grilli et al., 2011; Nouguier et al., 2014) indicate a yet too limited spatial extent of the measurement region. As a result, the application of such technology, in case of deterministic predictions in realistic sea states characterised by long waves in open sea, becomes difficult. Nevertheless, an extension of the LIDAR wave measurement range could allow this technology to become a possible alternative to wave radar.

Once wave elevation data are (assumed to be) available from a suitable wave measurement system, a phase-resolved propagation procedure can then be applied to perform a deterministic forecasting. The procedure is required to be fast if the use is intended for real-time applications. Furthermore it is required to have a prediction time horizon compatible with the operational needs. Particularly due to computational speed requirements, linear deterministic wave propagation models are often preferred (Hilmer and Thornhill, 2015), especially for intended uses in real-time applications, and different aspects of their implementation

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have been investigated in the past (Belmont et al., 2006, 2014; Blondel-Couprie and Naaijen, 2012; Connell et al., 2015; Naaijen and Blondel-Couprie, 2012; Naaijen et al., 2014; Naaijen and Huijsmans, 2008). Linear DSWP procedures usually consist of two main steps. First, in the *fitting step (FS)*, the wave elevation data are analysed in the measurement domain by means of Fourier decomposition techniques, either based on the DFT (Morris et al., 1998; Naaijen and Blondel-Couprie, 2012; Vettor, 2010) or on a least-squares approach (Connell et al., 2015; Naaijen et al., 2009; Vettor, 2010). Afterwards a linear propagation model is defined in the *propagation step (PS)*. The extensive use of the FFT, both in the FS and in the PS steps, is deeply discussed for short-crested sea applications by Blondel-Couprie and Naaijen (2012) and Naaijen and Blondel-Couprie (2012). Different implementations of linear fitting and propagation procedures, still based on a Fourier analysis, are presented by Abusedra and Belmont (2011) and Belmont et al. (2006).

Nonlinear phase-resolved wave propagation models have been proposed by several authors (e.g. Blondel et al., 2010; Blondel-Couprie et al., 2013; Nouguier et al., 2014; Wu, 2004; Zhang et al., 1999a,b; Yoon et al., 2016). The main issue of these techniques is represented by the costly fitting/initialization step. In fact, in general, the measured wave elevation data has to be pre-processed before being actually available for the propagation model. The pre-processing step, and its consequent complexity, depend mainly on the nature of the available measurement and on the nonlinear model considered. The reconstruction of the initial conditions of the nonlinear wave model can require iterative procedures on the measured data (Zhang et al., 1999a,b) or data assimilation procedures as in Wu (2004), Blondel et al. (2010), Blondel-Couprie et al. (2013) or Yoon et al. (2016). In particular, the variational data assimilation procedure proposed by Wu (2004) and Blondel et al. (2010) can be considered as an optimization problem for the initial conditions of the model, with cost function defined as a suitable measure of the distance between the wave elevation given by the nonlinear model and the measured wave elevation data. Also Nouguier et al. (2014) used the minimization of a cost function, representing the average squared difference between measured wave elevation and wave model to be propagated, for the identification of the free parameters of the wave propagation model. Actually, such approach, in addition of being used for forecasting purposes, served also the purpose of wave elevation reconstruction procedure for LIDAR measurements (Grilli et al., 2011; Nouguier et al., 2014).

A common issue to all DSWP methods is related to the need of providing an estimation of the region where the deterministic prediction can be considered to be sufficiently reliable for the intended purposes. In fact, any DSWP procedure is inevitably affected by prediction errors with respect to the true wave elevation, which indirectly define the limits of application of this kind of procedures. One source of prediction error is the inherent limitation of the assumed propagation model which does not exactly represent the underlying wave elevation field. As a result, even when a propagation model perfectly fits the true wave elevation at some discrete sampling points in time and/or space, the predicted (or reconstructed) wave elevation at different locations in time and/or space will differ from the true one. In addition, in real applications, the wave measurements themselves are affected by measurement errors, which bring into the problem an additional source of uncertainty, an aspect which is often overlooked. This means that a key aspect of DSWP should be the capability of providing not only an estimation of the predicted wave elevation, but also some information regarding the prediction error. However, although the assessment of the prediction error is crucial for a consistent deterministic wave prediction, the problem is rarely addressed specifically. In this context, the idea of using brute force tools such as massive Monte Carlo simulations to estimate the expected prediction errors statistics is, in general, practically unfeasible due to the time consuming computations that eventually will go to detriment of a direct use in real-time applications. Therefore, concepts of faster and more direct application are required.

The most widespread concept related to the prediction performance of DSWP approaches is the so-called “Predictability Region”. The Predictability Region is considered to be the region of space and time where it is considered “possible” to predict the wave elevation, ideally without errors. It is therefore, originally, a binary concept, which split the time/space domain in a region where the prediction “is possible”, and a region where the prediction “is not possible”. In the past, a matter of discussion has been whether to use the group velocity or the phase velocity of the waves for the identification of the Predictability Region (e.g. Abusedra and Belmont, 2011; Edgar et al., 2000; Morris et al., 1998; Naaijen et al., 2014; Wu, 2004). For example, based on wave propagation considerations, Morris et al. (1998) selected the wave phase velocity for the determination of the region where the propagation of information, and the corresponding deterministic prediction, can be considered possible (see also Edgar et al., 2000), in case of long-crested seas. Instead, Wu (2004) used the wave group speed for the determination of the Predictability Region and further extended the concept to the case of short-crested seas. According to Wu (2004) (see also Naaijen et al. (2014)), the Predictability Region is defined using the group velocity of the fastest and slowest wave components of the considered sea spectrum. However, Abusedra and Belmont (2011) have shown that the use of wave group velocity cannot be completely justified, and they also challenged those previous justifications for such use which were based on asymptotic stationary phase approximation. The concept of Predictability Region has been further developed by Wu (2004) and Naaijen et al. (2014) with the introduction of the “Predictability Indicator”: a measure of the prediction capability at a generic point in time and space, given the sea spectrum. The Predictability Indicator takes into account the actual shape of the wave spectrum, and this represents an advance with respect to the standard Predictability Region, which, instead, accounts only for the (assumed) lowest and highest frequency limits of the spectrum. Naaijen et al. (2014) verified the Predictability Indicator method to be qualitatively consistent with Monte Carlo simulations for which every realization of the sea states is ideally measured and then propagated to create a reference statistics for ensemble analysis. The encouraging results showed by Naaijen et al. (2014) and the simple formulation of the method makes the Predictability Indicator an interesting tool for a more advanced, and potentially more precise, definition of Predictability Region compared to the original concept. However, the Predictability Indicator still lacks a consistent statistical background theory able to provide a clear probabilistic interpretation of the obtained quantitative values.

It is then useful to make a step forward in the definition of the concept of Predictability Region, with a view to more soundly account for the prediction error from a probabilistic perspective. To this end, a theoretical approach for providing a consistent probabilistic measure of prediction error for deterministic phase-resolved linear wave prediction models, is herein presented. The approach is based on the description of the sea as a Gaussian stationary stochastic process. The features of the fitting procedure and of the prediction model are naturally embedded in the formulation. Furthermore, the formulation takes into account, in an analytic way, the actual shape of the spectrum for long-crested and short-crested waves. On top of this, the proposed framework also allows taking consistently into account the possible presence of additional measurement noise.

The paper is structured as follows. First, the theoretical background is presented starting from the definition of the fitting model and then providing the definition of the prediction error as a stochastic process. The assessment of the ensemble variance of the error process leads to the natural definition of a Linear Estimator of Prediction Error (LEPrE), which accounts also for the possible contribution of measurement noise. A section then follows, containing three different simulated verification test cases, considering long-crested and short-crested sea states, to show how LEPrE can be used in identifying the level of prediction error. Reported results from the application of LEPrE are verified along with corresponding sets of Monte Carlo simulations. Eventually, some