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Short communication

An extension of a discrete-module-beam-bending-based hydroelasticity method for a flexible structure with complex geometric features

Xiantao Zhang[a,b], Da Lu[a,*]

[a] State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, China
[b] School of Civil, Environmental and Mining Engineering, The University of Western Australia, Australia

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ABSTRACT

The discrete-module-beam-bending based hydroelasticity method proposed by Lu et al. (2016) deals with the hydroelastic response of a flexible structure by discretising it into several rigid submodules which are connected by Euler-Bernoulli beam elements. A stiffness matrix is determined in terms of the geometrical and physical properties of the flexible structure for each beam element and it has an analytical form for simple geometric features such as a rectangular cross section unchanged along the longitudinal direction. However, the analytical stiffness matrix may not exist for a flexible structure with complex geometric features. In the present study, with the aid of finite element method, the discrete-module-beam-bending-based hydroelasticity method proposed by Lu et al. (2016) is extended to be applicable for a flexible structure with complex geometric features.

1. Introduction

Hydroelasticity is concerned with the deformation of flexible structures responding to hydrodynamic excitations and simultaneously the modification of the excitations due to the structural deformation. Traditional three-dimensional hydroelasticity theory based on mode superposition approach has been widely adopted for the dynamic response of flexible structures in waves (Senjanović et al., 2008). This method contains three steps: (1) Evaluation of the natural oscillation modes of the flexible structure; (2) hydrodynamic analysis for each mode and (3) Superposition of all modes together and solution of the coupled modal equation to obtain the hydroelastic response of the flexible structure.

Unlike the traditional mode superposition approach, Lu et al. (2016) proposed a discrete-module-beam-bending based hydroelasticity method. The underlying idea is outlined as follow. First, a continuous flexible structure is discretised into several rigid submodules. Multi-rigid-body hydrodynamics is adopted to obtain the hydrodynamic forces (wave excitation force, added mass force and radiation damping force) on each rigid submodule, which, together with the hydrostatic force and inertia force, comprises the total external force on each submodule. Then each submodule is simplified as a lumped mass at its center of gravity and adjacent lumped masses are connected by a beam element to account for effects of structural deformation. Finally, the equations of motion for a flexible structure in waves are established by considering the equilibrium of total external forces and structural deformation induced forces on each lumped mass. Some recent researches on application of this hydroelasticity method can refer to Sun et al. (2018), Wei et al. (2017), Xu et al. (2017), Zhang et al. (2018a, 2018b).

The flexible structures investigated in the above-mentioned researches have simple shapes with unchanged cross section in the longitudinal direction (a rectangular plate in Sun et al. (2018), Wei et al. (2017), Xu et al. (2017) and Zhang et al. (2018b). And an elliptic cylinder in Zhang et al. (2018a)). Thus for each beam element added between two adjacent lumped masses, an analytical stiffness matrix exists (see Appendix A). However, in reality, large ships or very large floating structures may have complicated geometric features and thus no analytical stiffness matrix exists for the beam element.

The aim of the present study is to extend the discrete-module-beam-bending-based hydroelasticity method proposed by Lu et al. (2016) to be applicable for a flexible structure with complex geometric features with the aid of finite element method. An outline of the underlying idea for the extension is given and some validations are provided.

2. Extension of the discrete-module-beam-bending based hydroelasticity method

2.1. Revisitation of the hydroelasticity method

The discrete-module-beam-bending-based hydroelasticity method proposed by Lu et al. (2016) is revisited here. For Lu's hydroelasticity

* Corresponding author.
E-mail address: nichenrugu@sjtu.edu.cn (D. Lu).

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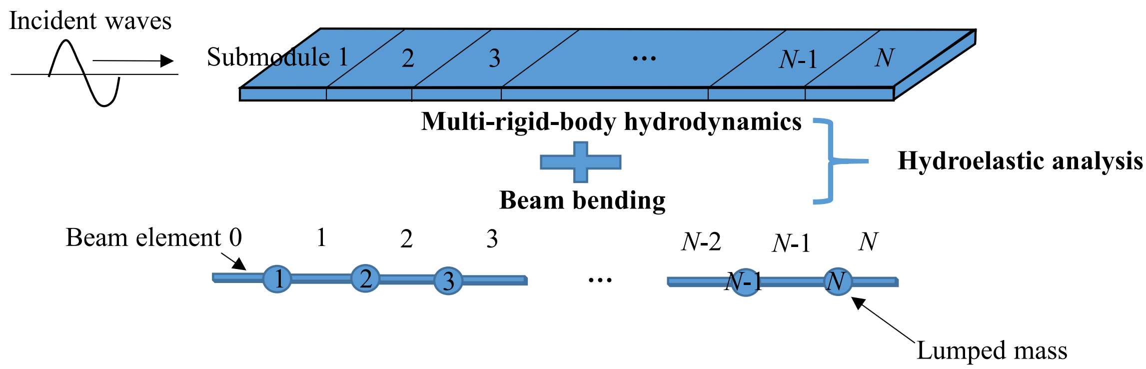

Fig. 1. Schematic of the discrete-module-beam-bending based hydroelasticity method.

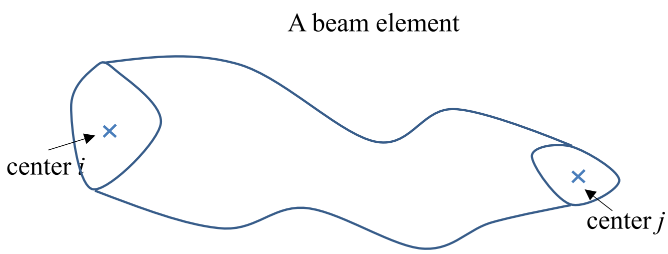

Fig. 2. A beam element with arbitrary shape of cross section and varied geometric features along the longitudinal direction.

method, a linear hydrodynamic approach is used and the solution is obtained in frequency domain. As shown in Fig. 1, a continuous flexible structure is first discretised into N rigid submodules. Then multi-rigid-body hydrodynamics theory can be used to obtain the wave excitation force $\mathbf{F}_E$, added mass force $\mathbf{F}_A = \omega^2\mathbf{A}(\omega)\boldsymbol{\xi}$, radiation damping force $\mathbf{F}_{\mathrm{Rd}} = i\omega\mathbf{B}(\omega)\boldsymbol{\xi}$ for all submodules. Here, ω is the wave frequency; $\boldsymbol{\xi}$ is the total displacement vector at the center of each submodule with the dimension 6N×1 (each submodule has 6 degrees of freedom); $\mathbf{A}(\omega)$ and $\mathbf{B}(\omega)$ are the added mass and radiation damping, respectively (with the dimension 6N×6N). The hydrostatic restoring force is $\mathbf{F}_{\mathrm{Hs}} = -\mathbf{C}\boldsymbol{\xi}$ with $\mathbf{C}$(whose dimension is 6N×6N) being the hydrostatic restoring coefficients. The inertia force of all submodules is $\mathbf{F}_{In} = \omega^2\mathbf{M}\boldsymbol{\xi}$, where $\mathbf{M}$(whose dimension is 6N×6N) is the mass matrix. The hydrodynamic formulations related to the calculation of the above-mentioned forces can be referred to Lu et al. (2016) and Zhang et al. (2018b).

The total external force exerted on the center of all submodules is

$$\mathbf{F}_{\mathrm{T}} = \mathbf{F}_{\mathrm{E}} + \mathbf{F}_{\mathrm{A}} + \mathbf{F}_{\mathrm{Rd}} + \mathbf{F}_{\mathrm{In}} + \mathbf{F}_{\mathrm{Hs}} \tag{1}$$

Subsequently, each submodule is simplified as a lumped mass at its center of gravity. And the adjacent lumped masses are connected by a beam element, the geometrical and physical properties of which are derived from the flexible structure. The force on lumped masses caused by structural deformation is $\mathbf{F}_{\mathbf{St}} = -\mathbf{K}_{\mathbf{St}}\xi$, where $\mathbf{K}_{St}$(6N×6N) is the stiffness matrix of the entire structure and it is given by overlaying the beam element stiffness matrix $\mathbf{K}_{\mathrm{E}}$ according to the standard process of finite element method. More details can be referred to Zhang et al. (2018a, 2018b).

Finally, the equations of motion of a flexible structure in waves are established in frequency domain by considering the equilibrium of forces for each lumped mass,

$$\{-\omega^2(\mathbf{M} + \mathbf{A}(\omega)) - i\omega\mathbf{B}(\omega) + (\mathbf{C} + \mathbf{K}_{\mathrm{St}})\}\boldsymbol{\xi} = \mathbf{F}_E \tag{2}$$

By solving Eq. (2), the displacement at the center of gravity of each submodule is obtained. Then the beam bending theory can be applied to solve the structural deflection, shear force and bending moment.

2.2. Extension of the hydroelasticity method

If the flexible structure has a simple shape, an analytical expression of the beam element stiffness matrix $\mathbf{K}_E$(the dimension is 12×12) can be given. For example, for a uniform beam element with a rectangular cross section, the expression of $\mathbf{K}_E$ is given in Appendix A. However, for a flexible structure with complex geometric features, we can never obtain such an analytical expression for the beam element stiffness matrix. As a result, the finite element approach will be adopted to extend the present hydroelasticity method. When it comes to the finite element approach, we mean that the beam between two adjacent lumped masses is discretised into a large number of small elements (a standard discretization process used in the finite element analysis).

We consider a beam element with arbitrary shape of cross section and varied geometric features along the longitudinal direction shown in Fig. 2. This beam element is bounded by the center of gravity of the ith and jth submodules. The extension of the approach actually includes two steps: (1) to represent the physical variables (displacement and forces) of all the nodes of the beam element by the physical variables of nodes on the cross section of the beam element; and (2) to represent the physical variables of nodes on the cross section by the ones at the center of cross section of the beam element.

The finite element method (FEM) is applied to the beam element for calculation of the overall stiffness of the structure. Suppose that the beam element is discretised into a number of small elements with the number of nodes n. Each node has three components of displacement and force, which is defined as $\boldsymbol{\xi}_p^* = \left(\xi_{xp}^*\ \xi_{yp}^*\ \xi_{zp}^*\right)^{\mathrm{T}}$ and $\mathbf{F}_p^* = (F_{xp}^*\ F_{yp}^*\ F_{zp}^*)^{\mathrm{T}}(p = 1, 2, \ldots, n)$. The force is related with the displacement by the spatial stiffness matrix $\mathbf{K}^*$(the dimension is $3n\times3n$) as

$$\mathbf{K}^*\boldsymbol{\xi}^* = \mathbf{K}^*\begin{bmatrix}\boldsymbol{\xi}_1^*\\ \boldsymbol{\xi}_2^*\\ \vdots\\ \boldsymbol{\xi}_n^*\end{bmatrix}_{3n\times1} = \mathbf{F}^* = \begin{bmatrix}\mathbf{F}_1^*\\ \mathbf{F}_2^*\\ \vdots\\ \mathbf{F}_n^*\end{bmatrix}_{3n\times1} \tag{3}$$

where the spatial stiffness matrix $\mathbf{K}^*$ can be obtained using the software ANSYS (2013).

In Eq. (3), the nodes are spatially distributed within the beam element. This means that there are nodes both on the two end cross sections of the beam (two cross sections where the centers i and j are located) and in the inner space of the beam. So the first step is to represent the physical variables (i.e. displacements and forces) of all the nodes of the beam element by the physical variables of nodes on two end cross sections of the beam element, i.e. to establish the stiffness matrix of the nodes on two end cross sections, $\overline{\mathbf{K}}_{\mathrm{B}}^*$. This process is quite similar to the static condensation process in finite element method (Wilson, 1974). Eq. (3) can be modified as