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A new residue-based dynamic analysis method for offshore structures with non-zero initial conditions

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ABSTRACT

Different from the traditional time-domain methods, which usually employ a step-by-step procedure for response estimation of structures with non-zero initial conditions, a new time-domain response estimation method is proposed by using separated residues corresponding to original external loadings and non-zero initial conditions, to provide a more efficient algorithm for offshore structures with non-zero initial conditions. The key of the proposed method is that responses of the system are divided into three contributions in the Laplace domain: the first comes from the original external loadings, the second from the initial displacements, and the last from a simultaneous combination of initial displacements and velocities. One theoretical development of the proposed method is that these three parts are all represented by separated residues that can be estimated by using the state-space model, which is also the actual reason why each part of the Laplace-domain responses of the system can be easily transformed back to the time domain in terms of the inverse Laplace transform. Compared with the traditional time-domain methods, responses from the proposed method are directly estimated by utilizing the inverse Laplace transform, which implies that estimated time-domain responses will be continuous in the time domain, so more accurate results can be expected. In addition, the proposed method avoids the procedure of a step-by-step estimation; therefore, better computational efficiency can also be predicted. Three numerical examples and one experiment are used to investigate the performance of the proposed method: the first is a single-degree-of-freedom (SDOF) system to illustrate the procedure, the second is a six-DOF system aiming at extending the proposed method to multiple-DOF (MDOF) systems, and the last is a typically studied engineering structure, i.e., a beam model, to show the potential engineering applications of the proposed method. Numerical results show that the proposed method not only can provide much more accurate time-domain responses compared with those from the traditional time-domain methods, even when the time step used is not so accurate, but also has better computational efficiency, e.g., in the third example, the traditional time-domain method takes 172.39 s during the estimation of responses with 10 s, while the proposed method takes only 13.01 s. Finally, an experimental fixed offshore platform conducted at the lab of Ocean University of China is used to demonstrate the proposed method.

1. Introduction

In mathematics and particularly in dynamic systems, an initial condition is usually defined as a value of an evolving variable at the initial time (typically denoted as $t = 0$) (Baumol, 1970). To carry out the design of structures subjected to dynamic loads, dynamic responses of the system are usually estimated under the assumption that the structure is initially at rest by employing either time domain or transformed domain methodologies (Clough and Penzien, 1993; Paz, 1997). However, the state of non-zero conditions is typically treated as fact for dynamic analysis of in-service structures, such as structural health monitoring or fluid-structure interaction analysis of structures.

Mathematically, the dynamic behavior of a system can be represented by a set of simultaneous second-order linear ordinary differential equations. For distributed dynamic systems, an appropriate discretization process, such as the finite element method, can be used to

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Nomenclature

a_r, b_r	The r diagonal values of $\Theta^T \mathbb{A} \Theta$ and $\Theta^T \mathbb{B} \Theta$	^{T}	Transpose operator
$\mathbf{C}$, $\mathbf{C}_{q,p}$	Damping matrix and its submatrix by deleting the p th row and the q column	β, γ	The Newmark integration parameters
$D(s)$	Determinant of a matrix	β_l^f, β_l^s, β_l^h	Residues of the system, the s term and the hybrid term, respectively
$\mathbf{f}(t)$	Original external loading in time domain	$\eta_l^h = \eta_l^s = \eta_l^f$	Poles of the system
$\mathbf{H}(\mathbf{s})$	Transfer function	θ_r, ζ_r	The r th eigenvector and damping factor
$\mathbf{K}$, $\mathbf{K}_{q,p}$	Stiffness matrix and its submatrix by deleting the p th row and the q column	$*$	The conjugate of a vector
$\mathbf{M}$, $\mathbf{M}_{q,p}$	Mass matrix and its submatrix by deleting the p th row and the q column	s	The Laplace variable
N_p	Number of components of an external loading	Δt	Time interval
$\mathbf{x}$, $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$	Displacement, velocity and acceleration of a system	$\mathbf{\Theta}$	Modal matrix with the state eigenvectors
$\mathbf{x}_0$, $\dot{\mathbf{x}}_0$	Non-zero initial displacements and velocities	λ_n	The n th pole of the external loading
$\mathbf{X}_f(s)$, $\mathbf{X}_d(s)$, $\mathbf{X}_h(s)$	Contributions of the external loadings, the s term and the hybrid term respectively	$\mathcal{L}$	The operator of the Laplace transform
		σ_q^h	The q th element of $\mathbf{M}\dot{\mathbf{x}}(0) + \mathbf{C}\mathbf{x}(0)$
		τ_t^h, τ_t^f	Zeros of the hybrid term and the transfer function, respectively
		ω_r, ω_{dr}	The r th natural and damped frequencies
		ν_n, χ_n	Poles and residues of Laplace responses caused by external loadings

represent them via discrete dynamic system models. In both differential equations in continuous time and difference equations in discrete time, initial conditions affect the value of the dynamic variables (state variables) at any future time. When the initial conditions of a system are non-zero, which are traditionally solved in the time domain based on a step-by-step numerical integration procedure, such as Central Differences, Newmark, Wilson theta, etc. (Bathe, 1996). But the required time-step resolution should be carefully determined and usually depends on the desired level of solution precision (Craig and Kurdila, 2006). During the determination of the time increment, which is often called the time interval, three factors must be considered: (1) the rate of variation of the applied loading, (2) the complexity of the nonlinear damping and stiffness properties, and (3) the period of vibration of the structure. In general, the time interval must be short enough to permit reliable representation of all these factors, which also means that reliable results can be achieved only when a proper time interval is used. Currently, there is a great deal of commercial software available based on time-domain approaches, including a great variety of strategies for linear and non-linear analysis, and there are excellent pre- and post-processing modules that enormously simplify the daily design activities; reviews of more recent developments can be found in Chung and Lee (1994)and Hulbert and Chung (1996), where developments regarding the optimization of numerical dissipation are presented. However, one serious limitation of time-domain methods is that, if a very short time interval is used, it may be difficult to obtain reasonable computational efficiency, especially while evaluating a lengthy response with small time steps and large numbers of degrees of freedom.

Non-zero initial conditions are seldom considered in the frequency domain by using traditional frequency-domain methods, which require that the loading be resolved into its discrete harmonic components by Fourier transformation. The corresponding harmonic response components are then obtained by multiplying these loading components by the frequency response function of the structure, and finally the total response of the structure is obtained by implementing the inverse Fourier transform. However, a major limitation of frequency-domain methods is that the computed response is a steady-state response, i.e., a response assuming that the initial conditions are all zero. By taking into account the initial conditions, Veletsos and Ventura (Veletsos and Ventura, 1984, 1985) introduced a discrete Fourier transform (DFT)-based procedure for calculating the transient response of a linear single degree-of-freedom (SDOF) system from its corresponding steady-state response to a periodic extension of the excitation. The procedure involves the superposition of a corrective, free-vibration solution that effectively transforms the steady-state response to the desired transient response. However, it requires the computation of the problem time-domain Green¡¯s function from known frequency-domain matrix transfer functions. Mansur et al. (Mansur et al., 2000; 2004) used the pseudo-force concept by taking into account the non-zero initial conditions in the DFT-based frequency-domain analysis of continuous media discretized by the finite element method (FEM), in modal coordinates or in both nodal and modal coordinates. In many cases, however, frequency-domain approaches are adequate, e.g., when the physical properties are frequency-dependent, when design requires the use of spectra, etc. Recently, Liu et al. (Liu et al., 2015; 2016, 2017) proposed a new frequency-domain method that can consider non-zero initial conditions. However, the early time steps of estimated responses always have some errors, because of the use of the inverse Fourier transform (IFT).

Besides frequency- and time-domain methods, dynamic responses of a system can also be solved in the Laplace domain, which solves second-order linear ordinary differential equations using the Laplace transform and normally consists of three steps: (1) the given ordinary differential equation is transformed into an algebraic equation, called the subsidiary equation, in the Laplace domain (Polking et al., 2006); (2) the subsidiary equation is solved by purely algebraic manipulations; and (3) the solution in Step 2 is transformed back, resulting in the solution of the given problem. The Laplace transform method is particularly useful if the forward and inverse transforms can be found directly in a table of Laplace transforms or can be converted to forms that can be obtained by the table lookup. Thus, traditional Laplace methods have been limited to analytical operations for simple forms of input functions. Hu and Liu (Hu et al., 2016) recently developed an efficient pole-residue method to numerically compute dynamic responses of multiple-DOF (MDOF) systems to arbitrary loadings; the key concept and development is regarding how to compute the poles and residues of the output from those of the input and system transfer functions, and the accuracy of the new method, in theory, is higher than that of any time-domain approach. The limitation of reference (Hu et al., 2016) is based on the assumption that the system is initially at rest, i.e., only zero-valued initial conditions can be handled.

This paper aims to develop a new time-domain response estimation method for structures with non-zero initial conditions. Specifically, we expect that estimated responses from the proposed method are not limited to the chosen time interval, and that the proposed method has better computational efficiency. To demonstrate and investigate the performance of the proposed method, three numerical examples and one experiment are going to be employed: the first is a SDOF system to illustrate the procedure, the second is a six-DOF system to extend the proposed method from SDOF to MDOF systems, and the last is a 15-element cantilever beam to show the potential applications of the