Ocean Engineering 161 (2018) 205–220

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Semi-analytical solution to the second-order wave loads on a vertical cylinder in bi-chromatic bi-directional waves

Peiwen Cong [a,b,*], Wei Bai [c], Bin Teng [a], Ying Gou [a]

[a] State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian 116024, China
[b] Department of Civil and Environmental Engineering, National University of Singapore, 117576, Singapore
[c] School of Computing, Mathematics and Digital Technology, Manchester Metropolitan University, Chester Street, Manchester M1 5GD, UK

ARTICLE INFO

Keywords:
Wave force
Semi-analytical solution
Bi-directional waves
Bi-chromatic waves
Second order

ABSTRACT

A complete solution is presented for the second-order wave loads experienced by a uniform vertical cylinder in bi-chromatic bi-directional waves. The solution is obtained based on the introduction of an assisting radiation potential without explicitly evaluating the second-order diffraction potential. The semi-analytical formulation for calculating the wave loads is provided and an efficient numerical technique is developed to treat the oscillatory free-surface integral that appears in the force formulation. After validating the present solution by comparing with the predictions based on other methods, numerical studies are conducted for different combinations of incident wave frequencies and wave headings, and the influence of frequencies and headings of dual waves on the second-order wave loads is investigated. In addition, by expressing the second-order wave loads in a power expansion with respect to the wave frequency difference and wave heading difference which are both assumed to be small, approximations on the calculation of wave loads are developed. The accuracy of different approximations is assessed by comparing the approximate results with those based on the complete solution.

1. Introduction

In the offshore environment, the action of water waves is the primary source of external loads that need to be considered in the design of offshore structures. In the framework of potential flow theory, the perturbation procedure provides a powerful tool to investigate wave-body interaction problems, by which the linear, second-order and even higher-order models have been derived and implemented successfully in the past. The interaction of waves with arbitrary three-dimensional bodies can be in principle simulated numerically by the boundary element or finite element method. Although advances have been achieved in the numerical techniques associated with these approaches, the intensive computation still requires considerable amounts of CPU time and consumes large amounts of memory. In this regard, some researchers have considered idealized geometries, such as a circular cylinder (either bottom-mounted or truncated) and cylinder array, to approximate ocean structures and employed the analytical and semi-analytical approach to evaluate the hydrodynamic loads.

So far, the solution of the first-order wave-body interaction problem has progressed with great success and analytical solutions for fundamental geometric structures have been explored by many researchers. Examples include Garrett (1971), Yeung (1981), Kagemoto and Yue (1986), Linton and Evans (1990), Yılmaz and Incecik (1998), Wu et al. (2006), Siddorn and Taylor (2008), Zheng and Zhang (2015), Liu et al. (2016) and Göteman (2017). In an irregular sea, consisting of a superposition of regular wave components, second-order high- and low-frequency hydrodynamic forces arise at the sum and difference frequencies of the constituent linear waves. These non-linear wave loads can play an important role in exciting some important phenomena, such as slow drift and springing (Petrauskas and Liu, 1987; Eatock Taylor and Kernot, 1999). Therefore the second-order interaction between waves and structures has also attracted continuous attention from the researchers. For example, by utilizing the so-called indirect method (Lighthill, 1979; Molin, 1979) which is based on the introduction of an assisting radiation potential to calculate the second-order wave loads without explicitly evaluating the second-order diffraction potential, semi-analytical formulations for the second-order wave force applied on fundamental geometric structures have been presented by Eatock Taylor and Hung (1987), Abul-Azm and Williams (1988, 1989), Ghalayini and Williams (1991) and Moubayed and Williams (1995). On the other hand, more direct methods including the second-order potential itself were adopted in Kim and Yue (1990), Chau and Eatock Taylor (1992), Huang

* Corresponding author. State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian 116024, China.
E-mail address: pwcong@dlut.edu.cn (P. Cong).

https://doi.org/10.1016/j.oceaneng.2018.04.094
Received 18 December 2017; Received in revised form 21 March 2018; Accepted 29 April 2018

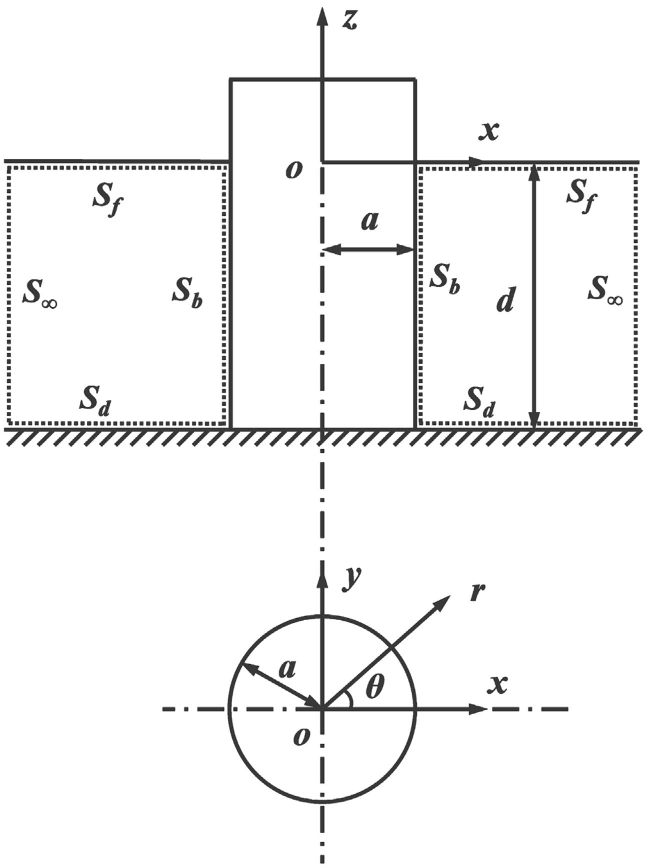

Fig. 1. Definition of the coordinate system.

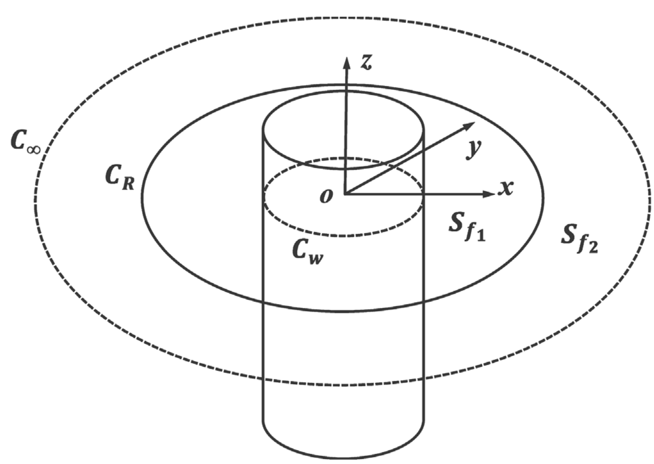

Fig. 2. Partition of the computational domains on the free surface.

and Eatock Taylor (1996), Teng and Kato (1999), Malenica et al. (1999).

The previous studies on the second-order wave diffraction primarily concern the action of unidirectional waves. However, to ensure a reliable design of offshore structures, it is of great demand to better understand the characteristics of the second-order wave-body interaction with respect to the wave directional spreading. If all the wave components in directional seas are assumed to be independent, the wave forces can be obtained from the superposition of directional component waves. However, Eatock Taylor et al. (1988) indicated that this kind of superposition may not yield reliable results if the second-order effects are included. Therefore, some researchers developed the design methods which

Table 1
Convergence test on the dimensionless sum- and difference-frequency surge forces, $f^{\pm}_{jl,\,x}$, on a vertical cylinder with varying N ($M=15$, $d/a=4$, $\beta_j=\pi/4$ and $\beta_l=0$).

(*a*) Sum-frequency

$(\nu_j a, \nu_l a)$	(1.2, 1.0)	(1.4, 1.0)	(1.6, 1.0)
$N=20$	(0.7355, 1.1099)	(0.7392, 0.7642)	(0.7488, 0.4227)
$N=50$	(0.7366, 1.1107)	(0.7407, 0.7650)	(0.7509, 0.4233)
$N=100$	(0.7367, 1.1108)	(0.7408, 0.7651)	(0.7510, 0.4233)
$N=150$	(0.7367, 1.1108)	(0.7408, 0.7651)	(0.7510, 0.4234)

(*b*) Difference-frequency

$(\nu_j a, \nu_l a)$	(1.2, 1.0)	(1.4, 1.0)	(1.6, 1.0)
$N=20$	(0.4930, –0.1316)	(0.4413, –0.2567)	(0.3984, –0.3820)
$N=50$	(0.4930, –0.1316)	(0.4413, –0.2567)	(0.3984, –0.3820)
$N=100$	(0.4930, –0.1316)	(0.4413, –0.2567)	(0.3984, –0.3820)
$N=150$	(0.4930, –0.1316)	(0.4413, –0.2567)	(0.3984, –0.3820)

Table 2
Convergence test on the dimensionless sum- and difference-frequency surge forces, $f^{\pm}_{jl,\,x}$, on a vertical cylinder with varying M ($N=100$, $d/a=4$, $\beta_j=\pi/4$ and $\beta_l=0$).

(*a*) Sum-frequency

$(\nu_j a, \nu_l a)$	(1.2, 1.0)	(1.4, 1.0)	(1.6, 1.0)
$M=3$	(0.7383, 1.1105)	(0.7421, 0.7635)	(0.7517, 0.4200)
$M=10$	(0.7367, 1.1108)	(0.7408, 0.7651)	(0.7510, 0.4233)
$M=15$	(0.7367, 1.1108)	(0.7408, 0.7651)	(0.7510, 0.4233)
$M=20$	(0.7367, 1.1108)	(0.7408, 0.7651)	(0.7510, 0.4233)

(*b*) Difference-frequency

$(\nu_j a, \nu_l a)$	(1.2, 1.0)	(1.4, 1.0)	(1.6, 1.0)
$M=3$	(0.4932, –0.1303)	(0.4415, –0.2539)	(0.3985, –0.3769)
$M=10$	(0.4930, –0.1316)	(0.4413, –0.2567)	(0.3984, –0.3820)
$M=15$	(0.4930, –0.1316)	(0.4413, –0.2567)	(0.3984, –0.3820)
$M=20$	(0.4930, –0.1316)	(0.4413, –0.2567)	(0.3984, –0.3820)

include both the second-order effects and wave directionality to investigate the properties of the second-order hydrodynamic loads induced by unidirectional waves. Kim (1992) developed a numerical model to predict the second-order difference-frequency wave forces on a large three-dimensional body in multi-directional waves based on the boundary integral equation method; Kim (1993) extended the asymptotic solution of the second-harmonic potential originally developed by Newman (1990) for the unidirectional wave to the multi-directional wave, and approximately evaluated the second-harmonic vertical forces on arrays of deep-draft vertical circular cylinders in monochromatic bi-directional waves. Based on the force formula on slender bodies proposed by Rainey (1989), Kim and Chen (1994) developed a slender-body approximation for the second-order difference-frequency wave force. Vazquez (1995) combined the boundary element method and indirect method to develop a solution for the second-order hydrodynamic loads on ocean structures in bi-chromatic bi-directional waves. Renaud et al. (2008) extended the middle-field formulation to the cases of bi-directional incident waves and performed calculation of wave drift loads and low-frequency loads on a LNG carrier.

In this study, the second-order interactions of plane bi-chromatic bi-directional incident waves with a vertical cylinder are considered, which is not well understood so far, but closely relevant to the design of marine structures in realistic ocean conditions. A complete solution for the second-order wave loads is developed based on the indirect method. The total second-order wave loads contain different constituent components which are related to the first-order interaction, the second-order incident potential and the second-order diffraction potential respectively. By utilizing Green's second identity, the force component associated with the second-order diffraction potential is expressed in terms of the free-surface and the body-surface integrals involving the first-order quantities and an assisting radiation potential. Evaluation of the oscillatory