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Vector form intrinsic finite element method for nonlinear analysis of three-dimensional marine risers

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ABSTRACT

Marine risers are dynamic sensitive systems and robust numerical models that describe their complex behavior are needed. The vector form intrinsic finite element (VFIFE) method is adopted to analyze the nonlinear behavior of marine risers with large deformations in three dimensional space. The method is based on vector mechanics theory and numerical calculation. The structural behavior is described by a simple physical model and strong-formed particle governing equations of motion. Three kinds of marine risers are presented to demonstrate the performance of VFIFE on analyzing marine risers under complex conditions. Hydrostatic and hydrodynamic loadings due to internal and external fluid acting on each element are considered. The modeling process is introduced in detail and the results are compared with the published literature. It is proved that the application of the VFIFE method in the nonlinear analysis of the three-dimensional marine risers is feasible and simply verified that loads outside of the plane have a more obvious impact on the risers, it is necessary to conduct three-dimensional analysis.

1. Introduction

With the continuous expansion of the human footstep, the exploitation of marine oil and gas has gradually entered the deep sea in which riser system is important as well as vulnerable connection part connected the sea floating devices to the submarine equipment (Chatjigeorgiou, 2008). As a consequence, the design, construction and installation methods of marine risers have become vital technical issues in the process of oil and gas exploration. According to DNV-OS-F201 (DNV, 2001), the marine risers are classified as top-tensioned risers and compliant risers by how floater motions should be absorbed by the riser system. Moreover, compliant risers can be not only flexible risers and steel catenary risers (SCR), but also pliant wave, lazy-wave, lazy-S, steep-wave riser. In virtue of their relatively low cost and good adaptability of floating and heaving motions, flexible risers and SCR are the preferred choices of risers for deep water oil and gas exploitation. Nearly a decade, the risers has attracted broad attention from scientific academia and industrial circles. Many researchers have conducted in-depth study on them, at the same time a number of the research works were published. Because of the similar structural mechanic characteristics, this paper would take both of the flexible riser and SCR as the examples of marine risers for study.

Jain (1994) gave the static analysis method of the overhang section of lazy-wave riser by finite difference (FD) method. Chatjigeorgiou (2008, 2010) proposed a finite differences solution method to analyze the numerical treatment of the dynamic equilibrium problem of 2D catenary risers, as well as the nonlinear dynamics problem of 3D catenary risers conveying fluid and subjected to end-imposed excitations. Webster (1977) conducted the earlier finite element analysis of the deep-sea moors and cable systems. Garrett (1982) presented a new three-dimensional finite element model of an inextensible elastic rod with equal principal stiffness. The model permits large deflections and finite rotations and accounts for tension variation along its length. Paulling and Webster (1986) improved the slender flexible rod model based on the finite element method of absolute node coordinate formulation, so that it can consider axial elongation. Chai and Varyani (2006) used the absolute coordinate formulation for the three-dimensional flexible risers for further analysis, taking into account the bending shear coupling, radial axial coupling and the effect of the flow. Nakajima et al. (1982) modified a new method to analyze the multi-component mooring system, and the method is motivated by the lumped mass method originally developed by Walton and Polachek (1959). Ghadimi (1988) derived the motion equations of flexible risers including nodal shear forces and bending moments in three dimensional space using the lumped mass discretization method.

It can be seen that the research and analysis methods currently used in marine risers mainly include finite difference method, finite element method and lumped mass method. These methods are characterized by

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mathematical analysis of the components. However this paper will take a new calculation method - vector form intrinsic finite element (VFIFE), also termed as finite particle method (FPM). VFIFE is a new numerical analysis method proposed by Ting et al. (2004). It is certainly an innovative concept of structural analysis. In this method, the structure is described as a collection of particles, which are connected by rod elements without quality and only withstand the internal force, and the motion of particles meet Newton's second law. The path unit is used to describe the motion state of the particle, and the virtual inverse motion is used to calculate the pure deformation. Substituting the mathematical model with the physical model, it does not require any integrated stiffness matrix, nor does it require an iterative solution to the governing equation. It can effectively deal with the geometric large displacement of space, non-linear and discontinuous material constitutive relations, structural and rigid body movement and their mutual coupling behavior and other complex situations. Wu et al. (2007), Wu and Ting, 2008) presented a large deflection analysis of membrane structures, in which a 4-node quadrilateral membrane element and a triangular membrane element were proposed to reveal that the proposed element based on an intrinsic model called VFIFE could go through the patch tests and possesses stable, convergent and accurate results. Yu and Luo (2009) proved the capabilities and accuracy of the FPM in their studies, using a motion analysis of deployable structures. Lee et al. (2007) found that the new developed method of VFIFE could be a promised method in analysis of the offshore template structures including its dynamic characteristic, large displacement motion and structural type through static and dynamic analysis of a typical marine riser.

In this paper, the VFIFE method is applied to analyze the three-dimensional marine risers, and the physical model of the risers is established. In the solution of the motion formula, the central differential method of explicit time integration is used. The calculation step includes two cycles: one is for particle position, the position of the current time is obtained by the position of last time calculated via the difference method; the other is for the element, the pure deformation is calculated by the virtual reverse motion, and the internal force of the element is calculated to obtain the particle force of the current position. The first cycle is used to solve the particle position of the next time point, the second cycle is used to solve the internal force of the next moment. Followed by reciprocating, it can obtain the state of each particle at every time point, to determine the whole configuration and internal force of the structure.

2. Fundamentals of vector form intrinsic finite element(VFIFE)

2.1. Point value description

Assuming that the rod (A− B) is a physical continuum structure and its shape is made up by an infinite number of points as presented in Fig. 1. Depending on the nature and precision of the problem, a limited number of points are selected among the infinite points, which are used to describe the shape and spatial position of the rod. The position of the other points between the spatial points can be represented by a set of successive normalized interpolation functions.

In this model, the rod is described by a finite number of points which have mass, also called particles. These particles with point values

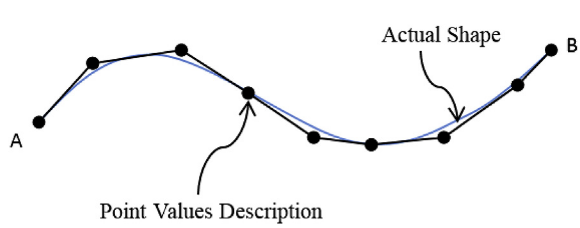

Fig. 1. Physical schematic of the description.

are connected by rod element which has no mass, and all internal forces and external forces are added to the particles. Therefore, the equivalent mass m^{α} (the quality of the rod element divided equally to the particles) is substituted for the quality of the hypostatic part of the rod. Such as the particle j, the mass of which is defined as:

$$m_j = \left(M_j + \sum_{\alpha=1}^{n} m^{\alpha}\right)_j, \quad m^{\alpha} = \frac{1}{2}\rho l \tag{1}$$

where M_j is the mass added on the particle j. n is the number of rod elements connected to the particle j, m^{α} is the mass contributed from each rod element and assigned to particle j. And ρ is the mass per unit length of the rod element, l is the length of the rod element. In addition, the riser includes the internal flow, according to the actual situation, it need consider the quality of the flow.

Similarly to m_j, the moment of inertia matrix $\mathbf{I}_j$ also includes the moment of inertia $\mathbf{I}_m$ contributed from spatial particle itself and the equivalent moment of inertia $\mathbf{I}^{\alpha}$ from the hypostatic rod. $\mathbf{I}_m$ and $\mathbf{I}^{\alpha}$ are both 3×3 matrices.

$$\mathbf{I}_j = \left(\mathbf{I}_m + \sum_{\alpha=1}^{n} \mathbf{I}^{\alpha}\right)_j, \quad \mathbf{I}^{\alpha} = m^{\alpha}\mathbf{r}^2 \tag{2}$$

Where $\mathbf{r}$ is the gyration radius of the cross-section in the direction of the principal coordinates. The proof can be found in Ref (Ting et al., 2012).

2.2. The route description by time points

When the rod is subjected to external forces, the position vector of any particle is a function relating to time variable, commonly known as a path or time trajectory described by the particle (on a set of time points in continuous time)values. Assuming that the analysis period $t_a \le t \le t_b$ is a time slice of the time trajectory and it can be described by a set of governing equations, it is called the path unit (Fig. 2). The position vectors of the particle at time t_a, t, t_b are expressed as $\mathbf{X}_a$, $\mathbf{X}$, $\mathbf{X}_b$ respectively. The question needs to be described is how the particle converts from $\mathbf{X}_a$ to $\mathbf{X}_b$. The mechanics law that needs to be followed in the position transformation process:

(1) The law of motion. The position of the rod is represented by a set of particles, and the particles will change positions once subjected to force, that is, the displacement vectors. Displacement follows the motion formula.
(2) Hooke's Law. After the force added to the rod, there must be a change in shape, that is, the deformation or the relative

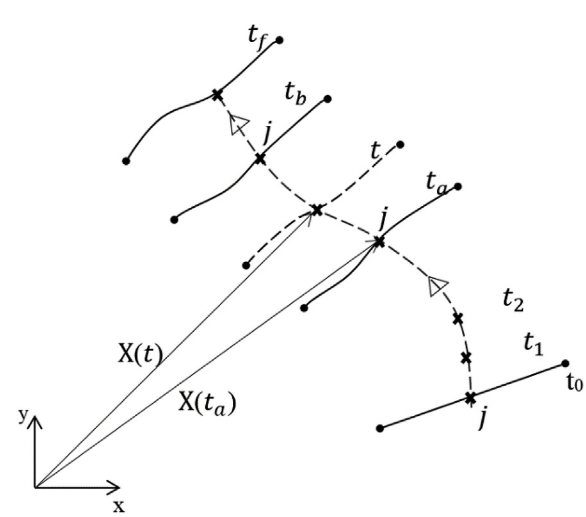

Fig. 2. Path unit.