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Time-frequency analysis of ship wave patterns in shallow water: modelling and experiments

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ABSTRACT

A spectrogram of a ship wake is a heat map that visualises the time-dependent frequency spectrum of surface height measurements taken at a single point as the ship travels by. Spectrograms are easy to compute and, if properly interpreted, have the potential to provide crucial information about various properties of the ship in question. Here we use geometrical arguments and analysis of an idealised mathematical model to identify features of spectrograms, concentrating on the effects of a finite-depth channel. Our results depend heavily on whether the flow regime is subcritical or supercritical. To support our theoretical predictions, we compare with data taken from experiments we conducted in a model test basin using a variety of realistic ship hulls. Finally, we note that vessels with a high aspect ratio appear to produce spectrogram data that contains periodic patterns. We can reproduce this behaviour in our mathematical model by using a so-called two-point wavemaker. These results highlight the role of wave interference effects in spectrograms of ship wakes.

1. Introduction

There is recent interest in applying time–frequency analysis as a tool for capturing various features of a ship wake. Such analysis involves performing many short-time Fourier transforms on a cross-section of a ship wake and visualising the resulting data as a spectrogram (Brown et al., 1989; Didenkulova et al., 2013; Sheremet et al., 2013; Torsvik et al., 2015a, 2015b; Wyatt and Hall, 1988). As this approach only requires measurements taken at a single stationary sensor as a ship sails past (Brown et al., 1989; David et al., 2017; Parnell et al., 2008), it offers a viable method for analysing ship waves in real-world conditions. This line of research has a range of potential practical applications in terms of quantifying the negative effects that a propagating wake wash will have when it interacts with a coastal zone (Torsvik et al., 2015b) or facilitating remote sensing and surveillance of unmonitored vessels.

Before now, the published studies on spectrogram analysis of ship wakes use experimental measurements taken from an open body of water (Brown et al., 1989; Parnell et al., 2008; Sheremet et al., 2013). As such, the surface height data involves a complicated combination of information from multiple ships and wind waves. The resulting interference has made it difficult for previous researchers to confidently attribute features of a spectrogram to various properties of a ship or its wake (Torsvik et al., 2015b). In response to this challenge, Pethiyagoda et al. (2017) used linear water wave theory together with a mathematical model to explain features of spectrograms of ship waves that are small in amplitude. This explanation includes a derivation of the linear dispersion curve which predicts the location of the colour intensity on the spectrogram when the ship in question is travelling through an infinitely deep body of water. Further, they used a simple weakly nonlinear theory and simulations of the fully nonlinear version of the model to identify features of a spectrogram that are due to nonlinearity. The comparison between these new theoretical results with data measured in a shipping channel in the Gulf of Finland is promising (Pethiyagoda et al., 2017); however, there is a need to extend the analysis to hold for finite-depth channels and to test the predictions against cleaner data gathered from controlled experiments.

In this paper, we extend the analysis in (Pethiyagoda et al., 2017) to apply to finite-depth channels. Ship wakes on a finite-depth fluid are interesting because they can be classified as either subcritical ($F_H < 1$, where F_H is the depth based Froude number) or supercritical ($F_H > 1$), with qualitatively different wave patterns forming for each case. For example, wakes for subcritical flows are made up of transverse waves and

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divergent waves, while wakes for supercritical flows contain divergent waves only. It is interesting that the wavelength of supercritical waves can vary greatly, and often contain some very long-wavelength waves which are potentially very damaging to shorelines in sheltered waterways, where such energetic waves do not occur naturally (Macfarlane et al., 2014). The difference between subcritical and supercritical flows has a noticeable effect on the linear dispersion relation which, as we show, directly affects the location of the high intensity regions on the spectrograms. We support our theoretical findings by comparing with experimental surface height measurements we have taken from a model test basin, eliminating the effect of environmental factors such as wind waves, currents or varying bathymetry.

Our study begins in Section 2 by presenting our mathematical model, which involves inviscid fluid flow past an applied Gaussian-type pressure on the surface. Here the pressure distribution is used as an idealised mathematical representation of a ship and the strength of the pressure (ε) acts as a proxy for the volume of water displaced by the ship. In Section 3 we extend the geometric argument for determining the linear dispersion curve presented by Pethiyagoda et al. (2017) to include the effects of a finite depth fluid. We observe the effect of changing the width of the Gaussian pressure relative to the water depth (δ), where the regions of high intensity favour higher frequencies along the dispersion curves for smaller pressure widths. Then, in Section 4 we validate the new linear and second-order dispersion curves against spectrograms generated from experimental data we collected from the model test basin at the Australian Maritime College. We observe trends in the experimental spectrograms for different sailing speeds and hull shapes, and provide possible explanations for these trends in terms of linear and nonlinear wave properties. Lastly, in Section 5, certain periodic patterns in the spectrograms for vessels with a high aspect ratio are explained by taking into account wave interference effects.

2. Mathematical model

2.1. Problem setup

In order to simulate a wake left behind a moving ship, we consider the idealised problem of calculating the free surface disturbance created by a steadily moving pressure distribution applied to the surface of a body of water of constant depth H. We suppose the pressure distribution is of a Gaussian type with strength P_0 and characteristic width L, and then formulate the mathematical problem in the reference frame of this moving pressure. We nondimensionalise the problem by scaling all velocities by the speed of the pressure distribution, U, and all lengths by U^2/g, where g is acceleration due to gravity. The fully nonlinear governing equations are then

$$\nabla^2\phi = 0 \quad \text{for} \quad -F_H^{-2} < z < \zeta(x,y), \tag{1}$$

$$\frac{1}{2}|\nabla\phi|^2 + \zeta + \varepsilon p = \frac{1}{2} \quad \text{on} \quad z = \zeta(x,y), \tag{2}$$

$$\phi_x\zeta_x + \phi_y\zeta_y = \phi_z \quad \text{on} \quad z = \zeta(x,y), \tag{3}$$

$$\phi_z = 0 \quad \text{on } z = -F_H^{-2} \tag{4}$$

$$\phi \sim x \quad \text{as } x \to -\infty, \tag{5}$$

where $\phi(x,y,z)$ is the velocity potential, $\zeta(x,y)$ is the free-surface height, $\varepsilon = P_0/(\rho U^2)$ is the dimensionless pressure strength, ρ is the fluid density and $\varepsilon p(x,y)$ is the pressure distribution. For the present study we will use the pressure distribution

$$p(x,y) = \mathrm{e}^{-\pi^2 F_L^4 \left(x^2+y^2\right)}, \tag{6}$$

where $F_H = U/\sqrt{gH}$ is the depth-based Froude number and $F_L = U/\sqrt{gL}$ is the length-based Froude number. We can also define the scaled width of the pressure distribution $\delta = L/H = F_H^2/F_L^2$. In this formulation, F_H is the parameter that measures the speed of the moving pressure, while the pressure strength ε provides a measure of nonlinearity in the problem (the regime $\varepsilon \ll 1$ is approximately linear).

Our use of the applied pressure (6) to act as a mathematical representation of ship is deficient in the sense that it does not allow for precise features of the ship hull to be described in any way. Despite this modelling simplification, it is common to use this idea when analysing ship wakes from a mathematical perspective (Darmon et al., 2014; Ellingsen, 2014; Li and Ellingsen, 2016; Pethiyagoda et al., 2015; Smeltzer and Ellingsen, 2017), and indeed we show that there are many advantages in this approach. To understand the analogy with real ship vessels, it is worth interpreting ε as a crude measure of the volume of water displaced by a vessel. Further, note that we can easily extend this model to include multiple pressure distributions, as we do later in Section 5.

2.2. Exact solution to linear problem

For moderate to large values of ε, the mathematical problem (1)–(6) is highly nonlinear. From a computational perspective, obtaining accurate numerical solutions is challenging as the problem is three-dimensional and the upper surface $z = \zeta(x,y)$ is unknown and must be solved for as part of the solution process. Progress with the numerical solution to this problem and related problems can made using boundary integral methods (Forbes, 1989; Pethiyagoda et al., 2014a, 2014b; Părău and Vanden-Broeck, 2002), for example.

On the other hand, for weak pressure distributions, $\varepsilon \ll 1$, the problem (1)–(6) can be linearised. The linearised version has the exact solution (Wehausen and Laitone, 1960)

$$\zeta(x,y) = -\varepsilon p(x,y) + \frac{\varepsilon}{2\pi^2}\int_{-\pi/2}^{\pi/2}\int_0^{\infty}\frac{k^2\tilde{p}(k,\theta)\cos(k[|x|\cos\theta + y\sin\theta])}{k - \sec^2\theta\tanh\left(k/F_H^2\right)}\mathrm{d}k\mathrm{d}\theta - \frac{2\varepsilon F_H^2 H(x)}{\pi}\int_{\theta_0}^{\pi/2}\frac{k_0^2\tilde{p}(k_0,\theta)\sin(k_0x\cos\theta)\cos(k_0y\sin\theta)}{F_H^2 - \sec^2\theta\,\mathrm{sech}^2\left(k_0/F_H^2\right)}\mathrm{d}\theta, \tag{7}$$

where $\theta_0 = 0$ for $F_H < 1$ and $\theta_0 = \arccos(1/F_H)$ for $F_H > 1$,

$$\tilde{p}(k,\theta) = \exp\left(-k^2/\left(4\pi^2F_L^4\right)\right)/\left(\pi F_L^4\right)$$

is the Fourier transform of the pressure distribution (6), $H(x)$ is the Heaviside function and the path of integration over k is taken below the pole $k = k_0(\theta)$, where $k_0(\theta)$ is the real positive root of

$$k - \sec^2\theta\tanh\left(k/F_H^2\right) = 0, \quad \theta_0 < \theta < \frac{\pi}{2}. \tag{8}$$

Note (8) is the linear dispersion relation for steady ship wave patterns which we discuss in some detail in Section 3.2.

Fig. 1 presents free-surface profiles calculated using the exact linear solution (7) for a subcritical Froude number, $F_H = 0.6$, and a supercritical Froude number, $F_H = 1.34$. In the subcritical case we see that the wave pattern is comprised of transverse waves that run perpendicular to the direction of travel and divergent waves that are oblique to this direction. On the other hand, as noted in the Introduction, the wave pattern for the supercritical case contains only divergent waves.

3. Spectrograms of finite-depth ship wakes

3.1. Computing a spectrogram

To compute the spectrogram data for a given signal, $s(t)$, we take the square magnitude of a short-time Fourier transform